Exploring Slope with

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STAIRS & STEPS

These STEM-inspired activities elicit the intellectual need to learn about rate of change and slope.

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Recent emphases on STEM initiatives indicate that educators must focus on "increasing STEM literacy so all students can think critically in science, math, engineering, and technology" (The White House 2012). Connections between STEM disciplines have already been described in *Principles and Standards for School Mathematics*, which implored that students "solve problems that arise in mathematics and other contexts" (NCTM 2000, p. 402). The Common Core State Standards corroborate that notion and suggest that mathematics programs should prepare students "to solve problems arising in everyday life, society, and the workplace" (CCSSI 2010, p. 7). As much as ever before, mathematics teachers are searching for ways to connect mathematics to real-life scenarios within STEM contexts.

Rate of change is one important mathematical concept found in middle-grades curricula that can be tied to other STEM areas. For students at this level, rate of change appears in the study of functions (CCSSI 2010; NCTM 2000). As students develop skill in proportional reasoning, they examine graphical representations of linear functions, learn to associate "slope" with "steepness" and rate of change, and develop skill in calculating slopes of lines.

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Slope and rate of change, however, do not exist only in math class. Engineering applications associated with ramps, staircases, and roller coasters rely on analyses of slope and rate of change. In science class, students learn about velocity and speed, each of which connects to slope and rate of change. Technology can be used to support students as they explore slope and rate of change in these and other contexts. Given these connections, slope is an ideal topic with which to bring STEM into the mathematics classroom.

This article presents a series of science- and engineering-related activities addressing rate of change that occurred in a middle-grades public school math class. As students worked through the activities, they developed an understanding of slope and rate of change within STEM contexts.

CONTEXT OF THE LESSON

The activities were developed as part of Expeditions in Science, Technology, Engineering, and Education through Mathematics (ESTEEM), a professional development program funded by the Virginia State Department of Education and held at George Mason University in Fairfax, Virginia. The program's goals were to develop teachers' understandings of—

1. connections within mathematics;
2. connections between mathematics and other STEM disciplines, including statistics; and
3. the pedagogy that can be used to support student understandings of these connections.

The first component of ESTEEM was a summer institute designed to develop mathematics teachers' content knowledge and thinking about mathematics pedagogy. The second component consisted of a lesson study (Lewis 2002) in which teachers worked together to develop a STEM-related mathematics lesson, then one teacher taught it to a group of students while the others observed and took notes on the lesson's impact on student learning. All teachers then debriefed, with help from the project faculty, to discuss the lesson's effectiveness. The result was a modified lesson plan.

> The motion detector technology allowed students to explore a mathematical concept (slope and rate of change) **WITHIN A SCIENCE CONTEXT** (speed).

One focus of the institute's activity was rate of change. As teachers solved a range of engineering, mathematical, and statistical problems, they considered the role of rate of change. In addition, teachers experimented with motion detectors and learned how they could be used to generate graphical representations of movement in time and distance graphs.

Two of the teachers, Bagshaw and Collins, decided to focus their lesson study on slope and rate of change. The lesson was implemented in Collins's eighth-grade class with sixteen students. The lesson's activities gave students opportunities to (1) use technology to explore the connections between rate of change/slope and speed and (2) work within an engineering context to examine connections between rate of change/slope and steepness.

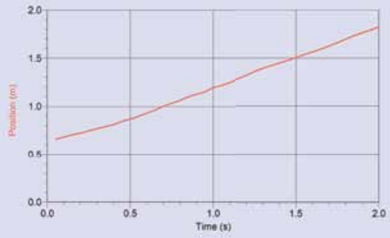
ACTIVITY 1: RATE OF CHANGE/SLOPE AND SPEED

The students were introduced to a motion detector and asked to move in front of it and look at the resulting graph. The graph plotted distance from the motion detector as a function of time. This was the first time these students had seen a motion detector, and various actions were tried. Several moved at different speeds (see **figs. 1a** and **1b**), one student spun in circles (see **fig. 1c**), and yet another simply stood still (see **fig. 1d**). After several students had taken a turn, the teacher asked what they noticed about their graphs. Students said that movement toward the motion detector resulted in a graph that "went down"; motion away from it resulted in a graph that "went up." The difference in steepness, as participants moved at slower and faster speeds, was also noted.

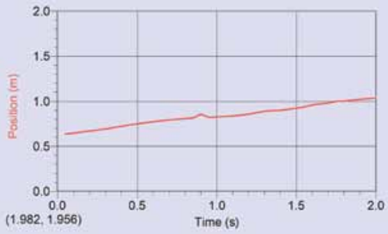
The teacher then directed the students' attention to an applet titled Football Distance Time Graph at http://www.sycd.co.uk/dtg/ (ASE 2011). After observing the applet, students saw a soccer player dribbling a ball toward the goal in a real game. As the player moved, a time and distance graph was created. Initially, the player moved at a constant speed, and students learned that a line on a time and distance graph represented such movement. As students continued to use the applet, they were asked to (a) match verbal descriptions of motion patterns to time and distance graphs and (b) match videos of movement to time and distance graphs.

These matching activities became more challenging as students were asked to consider graphs associated with the soccer players as they changed speed. The students enjoyed this activity and cheered when correct answers were chosen.

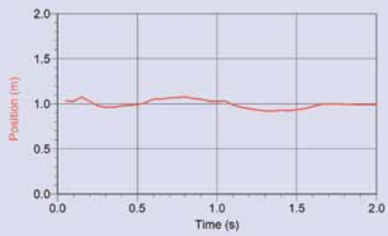
Fig. 1 Students produced these graphs after using a motion detector and moving in different ways and at different rates.



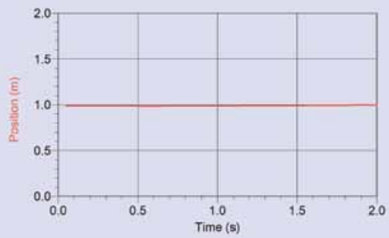
(a) Moving



(b) Moving slowly



(c) Spinning



(d) Standing still

Fig. 2 These questions led students through the steps of the staircase investigation.

INVESTIGATING STAIRS

Each group will measure two steps of a staircase. Measure the height and depth of each step using a ruler. Record your measurements. Sketch the staircase (two steps). Label each step with the appropriate measurement.

Questions

1. How could the measurements be changed to make the staircase steeper?
2. How could the measurements be changed to make the staircase less steep?
3. What is the vertical distance between the first and last steps?
4. What is the horizontal distance between the first and last steps?
5. What is the ratio of your answers to questions number 3 and number 4? How does this ratio compare with the ratio of the height and depth of a single step?

The use of technology to connect motion with a graphical representation was valuable. At no point during this section of the lesson did the teacher mention the word *slope.* The goal was to help students develop an intuitive understanding of the concept without getting bogged down in formulas or vocabulary. Because students were able to experience the creation of the graph with ongoing motion, they were able to connect the two. This technology allowed students to explore mathematical concepts (slope and rate of change) within a science context (speed).

ACTIVITY 2: RATE OF CHANGE/ SLOPE AND STEEPNESS

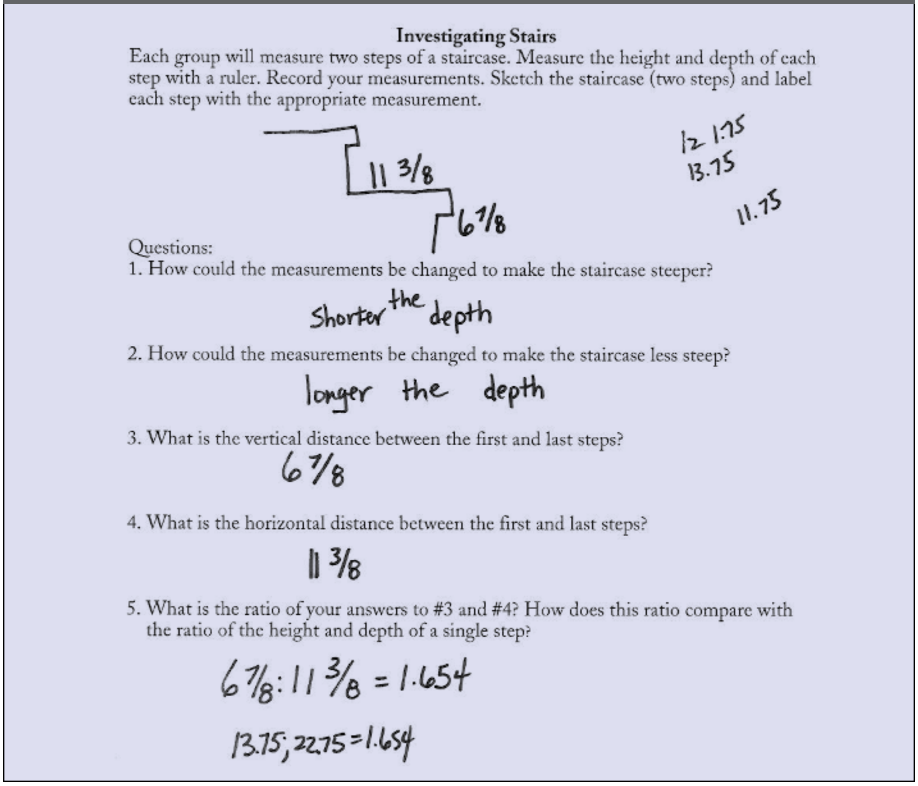
The Detective Slope lesson (NSA 2011) was the basis for activity 2. It was also similar to an item in the unit "Moving Straight Ahead" from *Connected Mathematics* (Lappan et al. 2004). The instructional progression aligned with the suggestions made by Lobato and Thanheiser (2002) for developing an understanding of the connection between steepness and slope.

Students were asked to look at pictures and compare the steepness of various hills and mountains. When asked how they might measure steepness, one student suggested measuring the height. The teacher asked if they should compare the height with something, and students responded, "Width." The teacher used hand motions to demonstrate how a tall but wide mountain would have different steepness compared with a tall mountain that was not as wide. The discussion transitioned to a consideration of stairs. One student asked if there was a connection to the term *slant*. The topic of wheelchair-access ramps was raised, and students questioned whether they should be steep or not. This discussion led nicely into an activity to investigate the steepness of a staircase (see **fig. 2**).

Students worked in groups to measure the height and depth of each of two steps in a hallway staircase. They were then asked how they could change the measurements to make the staircase more or less steep and to compare the ratio of the height and depth of two steps with one step. The students were engaged and excited about measuring the steps. Most students were able to correctly identify ways to change the steepness of the staircase. Some students suggested changing only one dimension (see **fig. 3**). Others suggested changing both dimensions (see **fig. 4**). Student work on these questions indicated an understanding of how the two dimensions work as a ratio to provide a "measure" of steepness.

Students were challenged by question 5, which asked about ratios (see **fig. 2**). Some students wrote the ratios without addressing their relationship (see **fig. 3**). Others wrote that the ratio for two steps is double that for one step (see **fig. 4**). This confusion was likely the result of having seen that the measurements for both the height and width doubled and assuming, therefore, that the ratio doubled. Only one student provided work to indicate

Fig. 3 Different approaches to changing the steepness of the stairs were described by students.



Investigating Stairs

Each group will measure two steps of a staircase. Measure the height and depth of each step with a ruler. Record your measurements. Sketch the staircase (two steps) and label each step with the appropriate measurement.

11 3/8

6 7/8

12 1.75
13.75
11.75

Questions:

1. How could the measurements be changed to make the staircase steeper?

Shorter the depth

2. How could the measurements be changed to make the staircase less steep?

longer the depth

3. What is the vertical distance between the first and last steps?

6 7/8

4. What is the horizontal distance between the first and last steps?

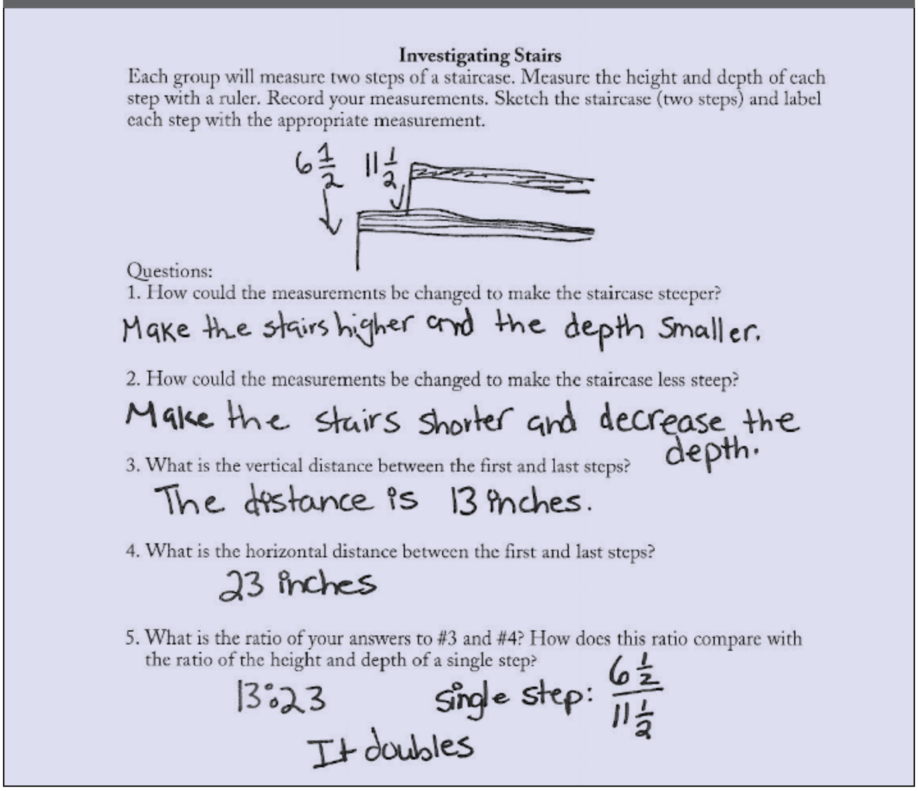
11 3/8

5. What is the ratio of your answers to #3 and #4? How does this ratio compare with the ratio of the height and depth of a single step?

6 7/8 : 11 3/8 = 1.654

13.75 ; 22.75 = 1.654

Fig. 4 Students had different perceptions of the consequences on the ratio of doubling the step sizes.



Investigating Stairs

Each group will measure two steps of a staircase. Measure the height and depth of each step with a ruler. Record your measurements. Sketch the staircase (two steps) and label each step with the appropriate measurement.

6 1/2 11 1/2

Questions:

1. How could the measurements be changed to make the staircase steeper?

Make the stairs higher and the depth smaller.

2. How could the measurements be changed to make the staircase less steep?

Make the stairs shorter and decrease the depth.

3. What is the vertical distance between the first and last steps?

The distance is 13 inches.

4. What is the horizontal distance between the first and last steps?

23 inches

5. What is the ratio of your answers to #3 and #4? How does this ratio compare with the ratio of the height and depth of a single step?

13:23 single step: $\frac{6\frac{1}{2}}{11\frac{1}{2}}$

It doubles

that the ratios were the same. In addition, because some students provided measurements in inches and others in centimeters, some concern evolved that the answers might be different. These issues elicited whole-class discussion that followed the group work.

As the class went over the answers to questions 3 through 5, the teacher asked students who had used inches in their measurements to share the ratios they found for one and two steps. The teacher then wrote the ratios

$$\frac{6.5}{11.5} \text{ and } \frac{13}{23}$$

on the board. Students who used centimeters were then asked to share their ratios. The teacher subsequently wrote the ratios

$$\frac{15.5}{28.5} \text{ and } \frac{31}{57}$$

on the board. With each pair of ratios on the board, the teacher asked the class whether they noticed anything. The students noted that, for both pairs of measurements (those done in inches and those done in centimeters), multiplying the first ratio by 2/2 results in the second ratio. When prompted, the students saw that the ratios were the same.

Using calculators, the students found that the quotients represented in inches and centimeters were actually quite close. This exploration led nicely into a discussion of precision of measurement as students recognized that some groups started measuring at the end of the ruler, rather than at the zero marking. Once students resolved the issue of measurement precision, they concluded that the quotient would be the same, whether measuring was completed in inches or centimeters.

Hallway stairs helped students think about rate of change and steepness as a ratio. After this staircase activity, students moved to the coordinate plane and began to look at the formula for slope. At this point, they began to use the term *slope*. The teacher was then able to connect steepness to slope from both geometric and algebraic standpoints, all within an engineering context.

DISCUSSION

Exploring the world and involving students in hands-on activities allowed critical thinking in STEM areas to develop naturally. The two activities helped develop students' thinking about rate of change and slope within STEM-related contexts. Activity 1 gave students opportunities to use technology, specifically the motion detector and the applet, to explore the connection between the graphical representation of rate of change and the scientific concept of speed. This activity not only provided opportunities to develop an understanding of the connections between mathematics and science but also prepared students for future study of physics and calculus.

Exploring the world and involving students in hands-on activities allowed **CRITICAL THINKING IN STEM** to develop naturally.

Activity 2 explored that same mathematical concept but within the engineering context of ramps and staircases. Concepts related to graphs and their characteristics permeate all fields of engineering, from civil through biomedical. Analyses of slopes, in particular, are necessary in designs from roller coasters to access ramps that are compliant with the American Disabilities Act. With exposure to hands-on activities like the staircase problem, students come to understand how math can be used to solve problems in engineering. They begin to develop an exploratory perspective of how to deal with engineering problems, which is valuable to future work in STEM fields.

REFLECTION ON IMPLEMENTATION

During the debriefing session, Bagshaw and Collins commented on

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the lesson's effectiveness. They believed that the discussions about change in "steepness" that emerged in both activities were valuable. As students worked with the idea of steepness, they developed a conceptual understanding of the rate of change and slope, which easily led into a discussion of formulas. These two teachers also noted that students who often sat quietly during class were engaged in the activities and class discussions. They were concerned, however, that the activities felt rushed; they plan to dedicate more time to them in the future.

When the lesson began, the staircase activity was presented first and the movement activity second. After a faculty member asked about reversing the two, the teachers decided that reversing them would help students connect the ideas more efficiently in that the technology activity supported an understanding of how motion can be represented on a coordinate plane.

While working with the graphs created using technology, students begin to connect the topics of speed and slope. As they transition to the staircase activity, they develop an understanding of slope and rate of change as a ratio. In working with ratios of lengths, students are able to transition to the coordinate plane to develop the formula to calculate slope. Comparing the steepness of one and two steps gives students opportunities to develop skill in proportional reasoning as they begin to understand constant rate of change.

The faculty member also suggested that graph paper could be used in the staircase activity to (a) demonstrate the equivalence of the ratio for one and two steps, thus helping to clarify the confusion that the ratio for two steps is double that for one step; and (b) support a transition to formulas on the coordinate plane. Bagshaw and Collins thus began to think about the possibility of using GeoGebra or The Geometer's Sketchpad® to engage students in a rate of change exploration.

> As students worked with the idea of steepness, they developed a **CONCEPTUAL UNDERSTANDING** of the rate of change and slope, which easily led into a discussion of formulas.

SLOPE IN CONTEXT

The activities presented here were highly successful in helping students develop an understanding of slope and rate of change within STEM contexts. After completing the activities, students were presented with time and distance graphs and asked to match the graphs with provided descriptions of motion. An analysis of their work indicated that students successfully completed this task. Another indication of understanding occurred when students were able to connect the work they did with staircases in a later lesson when they were asked to consider the formula for finding slope in the coordinate plane.

When teachers provide opportunities for students to explore a mathematics concept within STEM-related contexts, they are supporting the development of STEM literacy. This article presented one example of the many ways that teachers can bring science and engineering connections into math class. As teachers think of these ideas, they need to continue to share them so that we develop not only a mathematically literate but also a STEM-literate citizenry.

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