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Unlocking the Locker problem

The classic Locker problem is a rich benchmark problem that is familiar in both the school and college curriculum (Kaplan and Moskowitz 2000). It is a great mathematical puzzler that furnishes multiple entry points to access a variety of mathematical content in addition to encouraging multiple problem-solving strategies, multiple representations, critical thinking, justification, and proof—all of which reinforce the Process Standards (NCTM 2000). The development of flexible thinking, coherent representations, and adaptive strategies can also be encouraged and enhanced by technology, mindfully used.

This article describes how technology was used not as the learning goal but as a way to amplify the important mathematics in the Locker problem and solidify number characteristics while encouraging students to be curious mathematicians and future problem solvers. Classification of numbers is fundamental, foundational knowledge that every child must master. In the following sections, we describe a lesson that incorporated the Locker problem along with a technology tool to enhance student learning and focus on the mathematics that is unlocked through the Locker problem.

The lesson

Class began with a vocabulary warm-up of mathematical terms: *factors, multiples, prime numbers, composite numbers, evens, odds,* and *perfect squares.* At that time, students were asked to supply examples of factors and multiples and then determine the greatest common factor for two or more numbers and the least common multiple for two numbers. They were also asked to categorize numbers using the vocabulary terms. The teacher wanted students to categorize numbers into prime, composite, perfect squares, and even or odd. The class discussed the difference between factors and multiples and the number of factors for all prime numbers. The classroom teacher planned this intentionally to give her students access to the vocabulary that would be critical in making connections to the Locker problem. Following the vocabulary review, students received the problem (see the **sidebar** on **p. 322**).

As students began grappling with the problem, their teacher reminded them of the many available tools they had, including graph paper, colored chips, Unifix® cubes, playing cards, and so on. A student read the problem aloud and the class talked through the problem. The teacher asked the students to pay particular attention to the phrase *changes the state.*

The Locker problem

Following the vocabulary review, the teacher gave out the following problem to evaluate and assess student learning about number patterns and characteristics of odd, even, prime, composite, and perfect squares.

> In a long hall of Addalot School are 20 lockers numbered 1 to 20. In preparation for the beginning of school, the custodian cleans the lockers, resets the combinations, and closes the locker doors.
>
> When the Addalot students return from summer vacation, they decide to celebrate the beginning of the school year by working off some energy. Student 1 runs down the row of lockers and opens every door. Student 2 closes the doors of lockers 2, 4, 6, 8, and so on to the end of the line. Student 3 changes the state of the doors of lockers 3, 6, 9, 12, and so on to the end of the line. (This means the student opens the door if it is closed and closes the door if it is open.) Student 4 changes the state of the doors of lockers 4, 8, 12, 16, and so on. Student 5 changes the state of every fifth door. Student 6 changes the state of every sixth door, and so on, until 20 students have had a turn.
>
> When all the students have finished, which locker doors are open? Which locker doors are closed?

Acting it out

Initially, students acted out the word problem (see **figs. 1** and **2**). Five students came to the front of the class to "become lockers." All five stood with their backs to the class, representing closed lockers. A sixth student came forward and opened each locker by tapping all five students on the shoulder. All five students changed their state by turning around to face forward. Now all five lockers were open. Another student came forward and tapped the second locker and also the fourth. These two lockers faced backward again. The class noticed that only locker 2 and locker 4 changed, because they were multiples of two. This helped some students visually understand the different steps and terminology in the Locker problem.

Students worked first individually and then in small groups, using a variety of manipulatives and investigative methods. One of the first issues to arise was how to keep track of a locker's state. Some students used a chart, some used two-color chips, and some used playing cards. A second student concern was deciding what is important in the problem. Everyone knew to look for some pattern, but exactly what that pattern might be was unclear. Students received a sheet of questions to help guide their investigation (see the **sidebar** on **p. 323**).

Using technology to unlock the math

To facilitate the student learning process, the teacher presented a few slides from the Locker problem PowerPoint® animation at http://completegmu.org/locker.ppt. (Use the slideshow mode and hit ENTER to move to the next slide.)

Questions for the Locker problem

First working individually and then in small groups, students used manipulatives and investigative methods to keep track of the locker's state, to decide what was important in the problem, and to look for a pattern. They received the following questions to help guide their investigation:

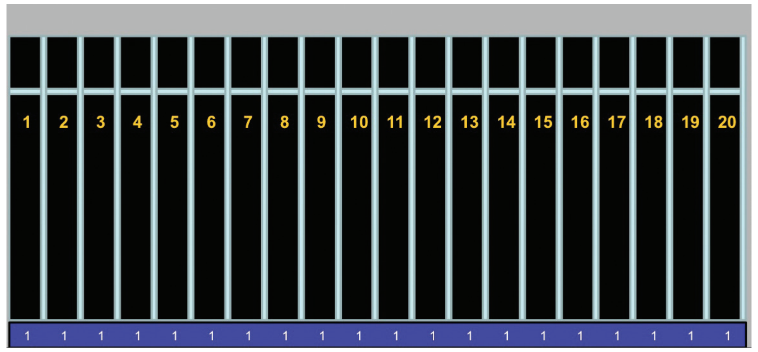
1. How are these locker numbers related or similar, and what are they called?
2. List the locker numbers that were touched by exactly two students.
3. List the locker numbers that were touched by an odd number of students.
4. List the locker numbers that were touched by an even number of students.
5. List the locker numbers that were touched by exactly three students.
6. List the locker numbers that were touched by four students.
7. How can you determine how many students have touched a specific locker?
8. State the first locker touched by both student 4 and student 6.
9. State the first locker touched by both student 5 and student 13.
10. State the first locker touched by both student 12 and student 30.
11. Which lockers are still open after the twentieth student is finished?
12. Which locker or lockers changed the most?

FIGURE 1

Before students arrived, the lockers were all closed.

FIGURE 2

Student 1 opened all the lockers.



The teacher asked, "What do you notice about all the lockers that are open?"

Students agreed, "All even lockers are closed, and all odd lockers are open." (See **fig. 3**.)

The teacher also asked the class to notice how often a locker's state was changed. Students continued to look for patterns as they kept track of the changes in the state of the lockers.

Note how the PowerPoint helps organize and keep track of the information. In addition to showing the state of a locker, open or closed, the numbers below the lockers are the number of times the locker has changed state or has been touched (see **figs. 1–5**). The tool helps students see numbers and number sense as a mathematical topic with order and a built-in system of relationships. These patterns can be used to enhance student thinking and conceptual understanding about fundamentals of number theory, including communicating about the classification of numbers as primes or composites, connecting with the concept of factors and multiples, understanding the development of common factors, and reducing fractions, thus extending the conceptual understanding from patterns to fractions.

Using such a simple technology tool to manipulate graphic displays aided the process of student learning in the classroom. The final state of the lockers, after all the students had finished, generated a lot of discussion that helped students answer several questions that the teacher had posed (see the **sidebar** on **p. 325**). For example, when the lockers displaying the number 2 at the bottom were all prime-numbered lockers (lockers 2, 3, 5, 7, 11, 13, 17, 19), they had changed state only twice, which validates the definition of a prime number. Before the simulation, students might have predicted that changing the state of every fifth locker would result in changes only to lockers 5, 10, 15, and 20. However, students must take into consideration that the state of the lockers had already changed four times, thus affecting the outcome of students 5's walk (see **fig. 4**). The lockers that displayed odd numbers at the bottom clearly were those of the perfect squares (1, 4, 9, 16), and these were the lockers that remained open after 20 students had walked past (see **fig. 5**). Lockers 12, 18, and 20 changed the most number of times, which brought up a discussion about the number of factors of any number. The configuration also helped the class discuss the misconception that the number 1 is a prime number. Extensions of the problem on greatest common factor and least common divisor were also discussed using the technology tool. The teacher connected the outcome of the technology to other representations that students had created, for instance, a graph representation (see **fig. 6**).

Students used such manipulatives as playing cards and two-color chips to simulate changing the state of a locker.

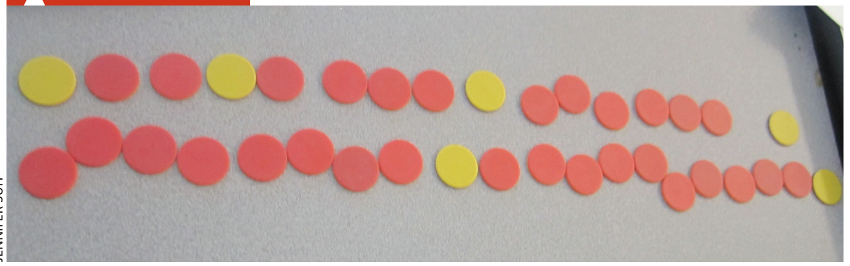


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FIGURE 3

Student 2 closed every other locker, starting from locker 2.

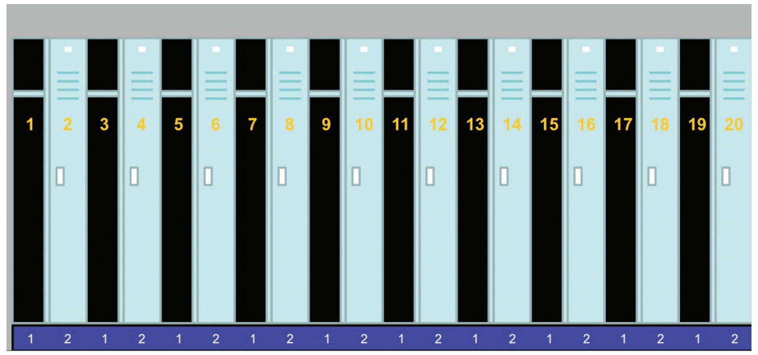
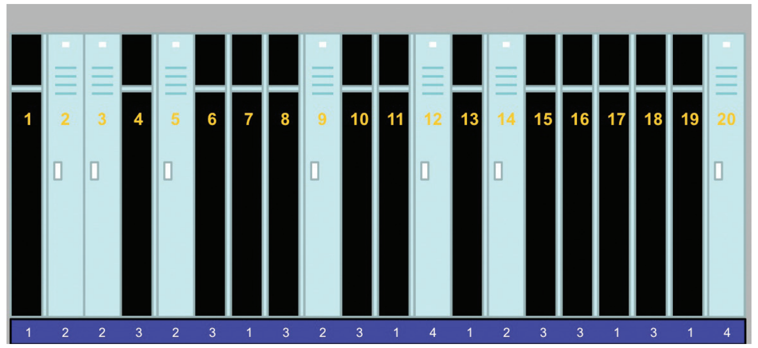


FIGURE 4

Student 5 started at locker 5 and changed the state of every fifth locker.



Concluding thoughts

We found that introducing the Locker problem with the PowerPoint animation engaged students in a rich mathematical problem and gave them access to analyzing patterns and learning about characteristics of unique numbers. The Locker problem was accessible to all the students, and the use of the models and acting-out strategies seemed to engage and motivate students. One hurdle with using physical manipulatives was that some students had difficulty keeping track of the changes in the state of the lockers and needed to use some form of a recording sheet. Once they began using graph paper to record the model and the actions done with the manipulative, students had more opportunities to look for and analyze patterns. The advantage of the technology was that it merged the animations in the problem scenario with the recording process that allowed students to become pattern seekers. Integrating technology effectively

The mathematics

This activity illustrates how technology was used to amplify the mathematics. Some of the essential mathematics that the teachers elicited from the mathematics discourse with students are summarized below.

- Every natural number is a divisor of itself, and 1 is a divisor of every natural number. This implies that every locker number greater than 1 has at least two positive divisors. For instance, every locker is opened on the first pass and shut on the pass where the student number equals the locker number.
- All numbers that are not perfect squares have an even number of positive divisors. To see this, consider any number N that is not a perfect square. Then every divisor D_1 of N has a corresponding divisor D_2 such that $N = D_1 D_2$. Since N is not a perfect square, $D_1 \neq D_2$. Since we can always pair a D_1 with a D_2, the total number of divisors can be counted in pairs. Thus the number of positive divisors is even. Hence every locker except those whose number is a perfect square has its state changed an even number of times: It gets changed and then changed back, or opened and shut again.
- All perfect squares have an odd number of positive divisors. Note that one can find pairs of divisors in perfect squares also. However, perfect squares will have a single leftover divisor (which is the square root of the perfect square) that cannot be paired with a different divisor. So the total number of divisors of perfect squares includes all the pairs and an extra divisor, which makes the total count odd. So the state of the lockers that are perfect squares is changed to an odd number of times (opened and left open).

FIGURE 5

This is the final screenshot of the state of the lockers after all twenty students opened and closed lockers.

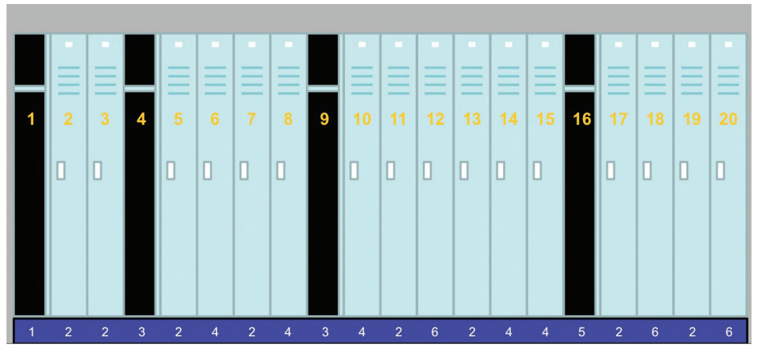
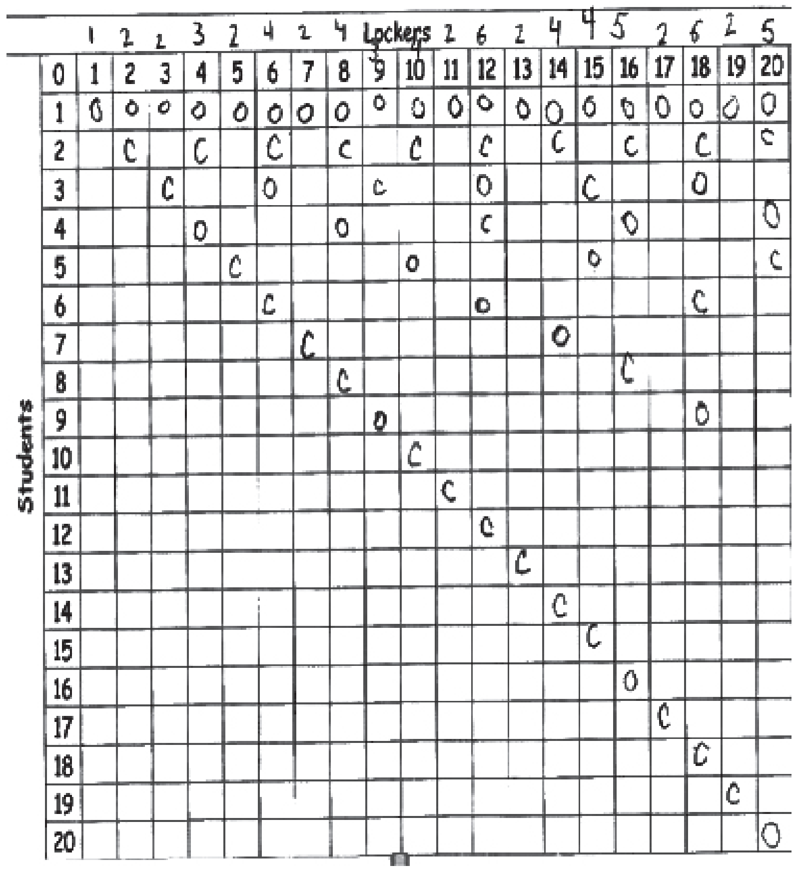


FIGURE 6

A graph was one student-created representation that the teacher connected to the outcome of the PowerPoint technology.



into specific mathematics investigations can give students access to advanced concepts and opportunities for higher-order thinking skills while allowing them to explore, test ideas, and in this case, unlock the secret to a rich mathematical problem. Other technology applets related to the locker problems can be found at the websites below.

http://connectedmath.msu.edu/CD/Grade6/Locker/index.html

http://www.math.msu.edu/~nathsinc/java/Lockers/

REFERENCES

Kaplan, Jerome, and Mark Moskowitz. 2000. *Mathematics Problem Solving*. New York: Triumph Learning.

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

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2013 Focus Issue:

Developing & Empowering Teacher Leaders

Are you a teacher leader? Whether you describe yourself as a math coach, math specialist, mentor, or the classroom teacher to whom everyone turns for support—you are a teacher leader.

A mathematics teacher leader wears many hats: mentor, data analyst, instructional collaborator, content and process facilitator, researcher, resource provider, observer, teacher, learner, and student. Producing students with proficient mathematical understanding requires teachers to have deep, diverse content knowledge, as well as be able to function in varied capacities to support and improve the teaching and learning of mathematics.

Teaching Children Mathematics (*TCM*) is interested in publishing articles that focus on developing and empowering teacher leaders in our mathematics communities. By highlighting these ideas about the diverse roles of teacher leaders and their impact on their school communities, the *TCM* Editorial Panel hopes to provide teachers and teacher educators with resources to inform and improve their educational practice and further their professional growth.

We invite authors to share their ideas and experiences surrounding the mathematics teacher leader in their professional community. Manuscripts that include photographs and samples of student work or dialogue are especially encouraged. The following list of topics and related questions is intended as a guide for authors. Manuscripts that address related issues are always welcome.

Consider the impact of a teacher leader on the school community and the school culture:

- How has the teacher leader influenced the school's professional learning community?
- In what ways has the teacher leader had an impact on student learning or achievement?
- How does the teacher leader mentor teachers or teacher teams?
- Give examples to support the impact a teacher leader has in fostering the development and growth of mathematical proficiency and understanding in a professional community.
- In what ways does a partnership with a teacher leader influence your own classroom practice?
- Share a personal story about moving into the role of teacher leader.
- Share case studies of your work as a teacher leader.

Consider lesson study and the teacher leader:

- What part does lesson study play in the role of the teacher leader?
- How does the teacher leader use lesson study to build communication and professional learning communities?

Limit your manuscript to 2500 words excluding references and figures; include figures and photographs at the end. On the cover page, state clearly that the manuscript is being submitted for the October 2013 *TCM* Focus issue, "Developing and Empowering Teacher Leaders." Author identification should appear on the cover page only.

Submit completed manuscripts by **July 31, 2012**, to *TCM* by accessing **http://tcm.msubmit.net**. Visit **www.nctm.org/journalsubmissions** for detailed manuscript preparation guidelines.

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