Mathematical Practices That Promote 21st Century Skills

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What skills will every student need to be successful in the twenty-first century? How do we prepare students to learn these skills? The Partnership for 21st Century Skills (P21), a collaborative effort among educators, businesses, and governments, is an organization that serves as a "catalyst to infuse 21st century knowledge and skills" (see **fig. 1**). Such knowledge and skills encompass life and career, media and technology, and learning and innovation.

The Learning and Innovation Skills portion of the framework emphasizes the 4Cs of critical thinking, communication, collaboration, and creativity. This article describes these skills in the context of the mathematics classroom and highlights teaching practices that can elicit each of the 4Cs to enhance student learning. **Table 1** summarizes P21's 4Cs; lists strategies that have been effective in promoting these essential skills; and links them, when applicable, to the Mathematical Practices outlined in the Common Core State Standards for Mathematics (CCSSM). This article will further expand on these strategies and share ways that the 4Cs can be incorporated into math lessons.

HOW TO SUPPORT THE SKILLS IN MATH CLASS

We describe a lesson that incorporates multiple Mathematical Practices and strategies that can be used to promote the 4Cs. We focus on the key practices that elicit many P21 skills, support the development of student thinking, and capitalize on the opportunity to learn important mathematics.

The task in **figure 2**, involving a comparison of MP3 music download plans, was presented in an eighth-grade classroom as a "Math Happening," a problem that is encountered in everyday life. The lesson focused on the 4Cs and the P21 theme of financial literacy. The mathematical goals for this problem included—

a. describing and representing relations and functions using tables, graphs, and rules;
b. relating and comparing tables, graphs, and rules as different representations of relationships; and
c. solving multistep linear equations and inequalities in one variable, solving equations (formulas) for a given variable, and applying these skills to solve practical problems.

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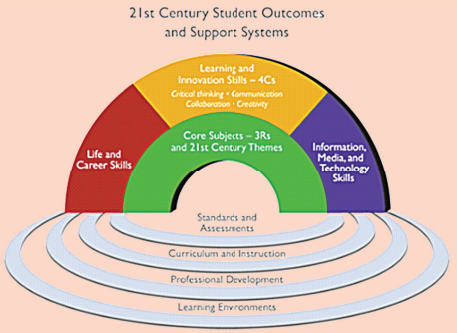
COMMUNICATION: A MULTIREPRESENTATIONAL APPROACH

Figure 3 illustrates how students analyzed three MP3 download plans in **figure 2** using a multirepresentational approach (creating a table, developing equations, and drawing graphs). This strategy promotes the fourth and fifth Standards for Mathematical Practice: "Model with mathematics" and "use appropriate tools strategically" (CCSSI 2010, p. 7). By analyzing the graph created with a graphing calculator, students noted that for a given number of downloads (i.e., input value), the line that was the "lowest" represented the best deal. The points of intersection represented where costs were the same.

Students made connections between the graph and a process of cost analysis based on the graphs. For some students, completing the table (the cost per plan, based on the number of downloads) proved most helpful in their analysis of which plan was best under various circumstances. Determining the linear functions ($c = d$, $c = 0.4d + 4$, and $c = 13$) was mathematically accessible for only a few advanced students. However, by

Fig. 1 This P21 diagram highlights the areas that can provide students with twenty-first century skills.



Source: http://www.p21.org

Fig. 2 This "Math Happening," choosing an MP3 purchase plan, provides a real-world context.

You have decided to use your allowance to buy an MP3 purchase plan. Your friend, Alex, is a member of i-sound and pays $1 for each download. Another friend, Taylor, belongs to Rhaps and pays $13 a month for an unlimited number of downloads. A third friend, Chris, belongs to e-musical and pays a $4 monthly membership fee and $0.40 a month per download. Each friend is trying to convince you to join his or her membership plan. Under what circumstances would you choose each of these plans and why? Present the best plan to your friends and convince them.

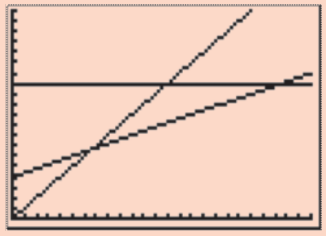
Table 1 P21's 4Cs align consistently with the Common Core's Mathematical Practices.

4Cs	P21	CCSSM and Strategies
Communication	Sharing ideas, questions, ideas, and solutions	"Model with mathematics" and "use appropriate tools strategically" (p. 7) through a multirepresentational approach with concrete manipulatives, tables, text, images, diagrams, and numbers
Collaboration	Working together to reach a goal—putting talent, expertise, and smarts to work	"Make sense of problems and persevere in solving them" (p. 6) by using group poster proof exercises for gallery walks
Critical thinking	Looking at problems in a new way, linking learning across subjects and disciplines	"Construct viable arguments and critique the reasoning of others" (p. 6) using the Strategy Venn Diagram and Questioning Prompt Cards
Creativity	Trying new approaches to get things done equals innovation and invention	Translate an everyday situation into a practical math situation with problem solving and problem posing using Math Happenings

Fig. 3 Students were asked to use multiple representations to communicate their solutions to the MP3 purchase plan problem.

Number of Downloads	i-sound	Rhaps	e-musical
0	0	13	4
1	1	13	4.40
2	2	13	4.80
3	3	13	5.20
4	4	13	5.60
5	5	13	6
6	6	13	6.40
7	7	13	6.80
8	8	13	7.20
9	9	13	7.60
10	10	13	8
11	11	13	8.40
12	12	13	8.80
13	13	13	9.20
14	14	13	9.60
15	15	13	10
16	16	13	10.40
17	17	13	10.80
18	18	13	11.20
19	19	13	11.60
20	20	13	12
21	21	13	12.40
22	22	13	12.80
23	23	13	13.20
24	24	13	13.60
25	25	13	14

(a) Table



(b) Graph

$y = x$ (i-sound)
$y = 13$ (Rhaps)
$y = 4 + 0.4x$ (e-musical)

(c) Algebraic formulas

comparing their work, many of the students were able to see how the tabular approach related to the algebraic equation and to the graph that some sketched for each plan. Through comparing and connecting representations and strategies, all students were able to see that at different numbers of downloads, the cost effectiveness of each plan varied.

CRITICAL THINKING USING PROMPT CARDS

To further stretch students' critical-thinking skills, teachers produced hypothetical scenarios in which students accessed data analysis and interpretation skills by using Questioning Prompt Cards (see **fig. 4**). These prompts allowed small groups to engage in a friendly debate, encouraged students to think critically about which plan was best under various circumstances, and made the mathematics meaningful. Students' small-group debates and verbal discussions contributed to a richer whole-class conversation about data analysis and how that skill helps them become better thinkers and smarter shoppers.

By posing some scenarios in which the cost benefit depends on the specific situation (see prompts 1–3), the teacher can capitalize on productive mathematical debates about which purchase plans appeal to the consumer, depending on how many MP3s the student buys in a month. This mathematical debate promotes the third Standard for Mathematical Practice, "construct viable arguments and critique the reasoning of others" (p. 6).

By posing these questions, students were also encouraged to think *creatively* in arguing for different plans in different situations and considering ways to advertise plans to consumers. In so doing, students were given the opportunity to employ two important P21 skills, namely, collaboration and communication.

Using the graph and the table, students could explain the break-even point and realize that at certain points two plans were equally cost effective. During this discussion, the teacher introduced the graphing calculator. Students were able to see how this technology tool could connect with their hand-drawn table and graph to interface with graphical, tabular, and algebraic expressions. They also learned to appreciate the efficiency of such technology use in math.

COLLABORATION WITH POSTER PROOFS AND GALLERY WALKS

The poster proof presentation strategy (see **fig. 5**) was designed to give students opportunities to communicate, collaborate, and reason through their ideas using multiple representations. *Principles and Standards for School Mathematics* (NCTM 2000) emphasizes that representations serve as tools for sense making, and for communicating, justifying, and connecting ideas by stating that representations allow students to communicate "mathematical approaches, arguments, and understanding to one's self and to others," and to "apply mathematics to realistic problem situations through modeling" (p. 67). Students were asked to create poster proofs in their small groups and to show different ways to represent their solution. The use of multiple representations promoted better communication of mathematics ideas. During gallery walks, students were asked to walk around the classroom, view their peers' poster proofs, and listen as they presented their reasoning.

Using group poster proofs and gallery walks are strategies that allow students to collaborate and build collective knowledge, solve problems, and defend their reasoning. CCSSM's first Mathematical Practice states that students should have opportunities to make sense of problems and persevere in solving them. Students should also learn to use and consider multiple approaches when solving problems. Promoting group poster proofs and gallery walks allowed students to develop twenty-first century learning skills while making sense of math. They further developed their ability

Fig. 4 Questioning Prompt Cards were used to support critical thinking.

1. If you buy fewer than 5 MP3s a month on average, which plan would you choose? Explain your answer.
2. If you buy between 10 and 22 MP3s per month on average, which plan would you choose? Explain your answer.
3. If you buy more than 22 MP3s per month, which plan would you choose? Explain your answer.
4. Which company has the best plan for you? Explain your reasoning.
5. Which representation—the table or the graph—helped you decide what plan is best? What information would you display in an advertisement if you were to sell one of the plans? Explain your reasoning.
6. What other real-life purchase plans could be analyzed in this way?

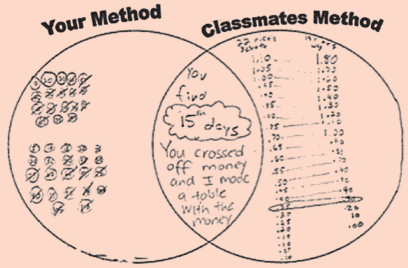
Fig. 5 A Venn diagram encouraged 4C analyses.

THE DONATION PROBLEM

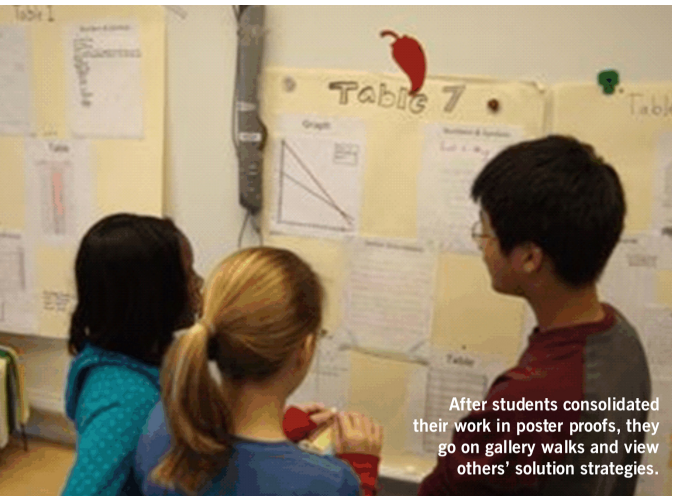
Nick and his older brother, Dave, decided to donate money from their piggy bank to their school fundraiser. There were just 18 dimes in Dave's piggy bank and just 22 nickels in Nick's piggy bank. They take one coin out everyday and put them in the school fundraising bank.

One day, they looked into each other's piggy bank. The younger brother's amount was greater than Dave's. When this happened, how many days had it been since they started their donation?

(a)



(b)



After students consolidated their work in poster proofs, they go on gallery walks and view others' solution strategies.

to make sense of problems by viewing and critiquing the approaches of others.

CREATIVE THINKING THROUGH MATH HAPPENINGS

Finally, to encourage creativity, students were asked to create a similar Math Happening that connected to the mathematics that they learned through the MP3 purchase plan problem. For example, problems related to *other* plans—cell phone, video game, or savings—were all possibilities. The task of writing a Math Happening encouraged not only problem solving but also the art of problem posing. Its related question required creativity, critical thinking, and the "doing and undoing" of a problem. After solving the given MP3 problem, students had to think back to all the discrete parts that were essential so that they could pose a comparable problem. In this case, it required students to think about a purchase plan with varying rates of change in the cost of the items.

A VENN DIAGRAM STRATEGY

Using an approach called Strategy Venn Diagram is another way to support communication and critical thinking (see **fig. 5a**). This multi-representational approach encourages students to solve a problem in a variety of ways. Once students approached a problem in one way, we asked each to develop a Venn diagram showing how he or she solved the problem. They then worked with a partner and compared strategies. Teachers and students can use the Venn diagram strategy to promote the third and fourth Standard for Mathematical Practice in CCSSM: "Construct viable arguments and critique the reasoning of others" (p. 6). Students also communicate their mathematical ideas by modeling the mathematics using various representations (concrete tools, picture and diagrams, tables, graphs, numbers, or verbal descriptions).

By collaborating on a Venn diagram, students compare and contrast strategies and think critically about how different strategies may be efficient or easier to understand. **Figure 5b** shows how two students approached the problem differently. Although one created a diagram and crossed off quantities and the other made a table, they both found the same answer. By allowing students to compare strategies, they learn to appreciate each other's perspective, connect their strategy to another's while evaluating its efficiency and clarity, and look for creative ways to reason. This Venn diagram strategy helps students engage all the twenty-first century skills while working together to find components to place in the union and the intersections of the Venn diagram.

TIPS FOR IMPLEMENTATION AND ASSESSMENT

One important aspect of encouraging the 4Cs in instruction and learning is heightening teacher and student awareness of these skills. Teachers need to be mindful of ways to elicit the 4Cs of communication, collaboration, critical thinking, and creativity by selecting rich problems, providing learning environments, and creating learning structures that amplify these skills.

First, selecting a rich problem as a Math Happening that can be solved in multiple ways with multiple mathematical connections can bridge the application of skills to real life. Spending time on a rich problem with a meaningful context may benefit students as they transfer the math skill to a real-life application.

Second, a learning environment that promotes twenty-first century skills does not happen overnight. A teacher must work with students in a classroom community to show what respecting others' ideas looks like and sounds like so that the mathematical debate is productive and moves the math agenda forward. To facilitate this type of mathematical dialogue, it is helpful to introduce debate sentence starters in this way: "I would like to offer another strategy . . ." or "I would like to build on my classmate's idea. . . ." In addition, the learning structures mentioned in this article should be introduced with several examples of modeling and sample student work. Just as one would not expect a new law student to be proficient at knowing how to debate or write legal decisions, one should not assume that students enter the class proficient at constructing mathematical arguments and critiquing the reasoning of others. These mathematical practices require the development of strategic thinking. By using an array of approaches, a teacher can support students to evaluate solution strategies based on efficiency, clarity, precision, and accuracy.

Another essential aspect of promoting twenty-first century skills in the math classroom is to help students understand the 4Cs and be able to self-assess and peer-assess across these four dimensions. Periodically, teachers can ask students to self-assess and peer-assess using prompts like those in **table 2** after a problem-solving task. These prompts allow students to reflect on how much an individual contributed to the learning experience and recognize the contributions of others. Assessing student skills and knowledge is essential to guide learning and provide feedback to both students and teachers about how well they are doing in reaching desired twenty-first century skills (Trilling and Fadel 2009). Formative assessment strategies such as self-assessment and peer-assessment can empower students to take more of an active role in improving their twenty-first century skills performance.

Table 2 These prompts gave students the opportunity to self-assess and peer-assess after a problem-solving task.

Assessing Your 21st Century Learning Skills in Mathematics				
Prompts to assess your 4Cs contribution	**Communication:** Did you share thoughts, questions, and solutions?	**Collaboration:** How did you work together to reach a goal using your knowledge, talents, and skills?	**Critical Thinking:** How did you solve this problem in new ways linking what you know?	**Creativity:** What new approaches did you consider to solve this problem, or did you invent a strategy that was efficient?
Self-assessment				
Peer-assessment				

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