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Being an environmentally friendly package engineer

Have you ever wondered who designed the cool triangular prism packaging for Toblerone chocolate? Did you know that people can make a whole career of package engineering? They design, develop, and produce packages and containers for all the goods we buy. A package engineer considers several factors: From a marketing perspective, a package engineer will want to attract consumers. From a production perspective, the engineer must also contemplate the material and assembly costs of the packaging as well as the process for producing the package. Another important consideration is the environmental factor, such as how eco-friendly the package materials are. The engineer's work also involves mathematics as he or she considers these factors to determine the most cost efficient (economical) and eco-friendly design so that materials are disposable, recyclable, biodegradable, and not wasted.

We present two different lessons that were planned by a group of teachers and math educators who are interested in incorporating STEM ideas into teaching important mathematical concepts. In both lessons, we focus on the concepts of measurement and geometry interconnected primarily with engineering.

The first lesson was taught in both a third-grade classroom and in a grade 5–6 multiage enrichment math classroom. We share the lesson ideas from both lessons so that other teachers might see how such a rich task as this can be differentiated and can be used to meaningfully teach multiple mathematical concepts across grade levels. Having a vertical team of teachers allowed us to discuss the important learning progression related to measurement and geometry.

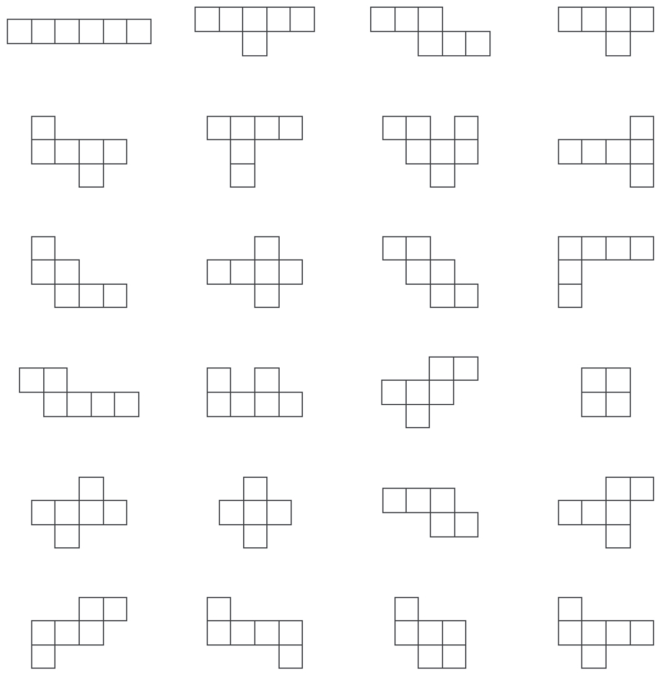
The second lesson was taught in the upper-grade geometry classroom, where students had an opportunity to investigate a problem from the perspective of a design engineer and to tackle a real packaging problem involving soccer balls. The lesson was presented to students in a sequence of increasingly challenging tasks that helped students become creative. Increasing the rigor in the problem through these tasks encouraged student engagement, which in turn helped them deepen their understanding and find practical solutions.

Becoming a package engineer

To begin the lesson, the teacher introduced the third graders to the career of a package engineer with a video clip from PBS Kids (http://pbskids.org/designsquad/video/package-design/). Then students received packages of different shapes to unpack. They opened the flaps and cut some edges to flatten the packages into *nets* (a two dimensional, or flat, figure that can be folded on its segments to form a three-

FIGURE 1

Foldable or not foldable? That is the question!
http://illuminations.nctm.org/ActivityDetail.aspx?ID=84



dimensional shape). They noticed that two-dimensional geometric shapes made up the three-dimensional solids and that some extraneous flaps were used with adhesives to assemble the packages. Then the "package engineers" were given the following quality-control task.

Part 1. Quality control

> You have been asked to review the design of these cube-shape packages. Please review the designs, and let us know which design passes inspection and which should be rejected.

First, students predicted which designs would fold into a box and which would not. Then they were to verify their predictions by cutting out the nets and folding them into a box. They were to write a report after the quality testing.

One student commented, "I didn't think a few of the nets would fold up into the cube, until I actually folded them. Some looked like they would have overlapped but did not!"

Students were especially surprised that a net that looked like a staircase folded into a cube.

Part 2. Designing a package

Next, students were given objects for which they were to design a package. They received large sheets of one-inch graph paper to create a prototype design. Many of the students used the ideas from the Foldable Net activity to plan their packages. One group created a cube and then realized that they had a lot of wasted space. They quickly planned and redesigned their package as a 4 × 4 × 2 rectangular prism to be more cost efficient and eco-friendly. To evaluate their design, students were asked to calculate the volume for their packages by filling them with inch cubes and covering the surface area by counting the inch squares on their net.

The design phase involved an iterative cycle. Students created their prototype after discussion and then altered and modified the design based on factors they considered, such as efficiency of space and cost, eco-friendliness, and design appeal. Students had to negotiate as they collaborated on the design. In the grades 5–6 classroom, we were able to provide the same scenario to explore packages, but we added more creative designs, including cylinders and triangular prisms, as an extension.

A STEM activity: Packaging soccer balls

Expanding on the Package Engineer problem, students designed boxes for various sports balls. During this lesson, students pretended to design boxes for the FIFA (who make the World Cup's soccer balls). To get an understanding of the process regarding how balls were produced, students watched a YouTube video on the production process of the Official Match Ball for the 2010 FIFA World Cup in South Africa (http://www.youtube.com/watch?v=zbLjk4OTRdI).

They were then told that the purchasing manager bought standard shipping boxes that measure 36 × 36 × 48 in. for all sports equipment to be shipped to the factory. With this information, students were asked to engage in four different tasks.

In the first task, students were to package soccer balls from 8 × 9 × 9 in. boxes into a larger standard shipping box. Using masking tape and a yardstick, students "drew" the box and the bottom layer of the shipping box when it is packed on the floor. From the bottom layer, students then had to determine how many total soccer balls would fit into the entire box. They had an

opportunity to compare this prediction with the number of soccer balls that would fit into the shipping box if calculated by using a simple formula for the volume of a rectangular prism.

In the second task, students were to package footballs that were in boxes that measure 6 × 7 × 12 in. into a larger shipping box. They were to take the same approach as in the first task, but the students soon figured out that extra gaps come up because of the fraction of the volumes. This discovery prompted them to think about and discuss how the choice of dimensions can make the problem interesting.

Following that task, students were given a design challenge: They were to package inflated soccer balls without boxes around them. The teacher brought in some inflated soccer balls to help students make reasonable predictions. The challenge was to determine how many soccer balls they could ship in the same large shipping box. They were then asked to compare the number of balls that this task yields to those from tasks 1 and 2.

Note how the sequence of tasks provided students with opportunities to develop their 21st-century skills (P21)—learning and innovation skills that emphasize the four Cs: communication, collaboration, creativity, and critical thinking. They used mathematical communication as they collaborated on the design challenges. They also used creative problem solving and critical thinking as they worked through the parameters and constraints in the design challenges. The tasks helped students engage in content with rigor and cognitive demand as well as create practical solutions through a project-based learning approach. Such a pedagogical approach shifts the responsibility of learning from the instructor to the student, helps to avoid rote memorization of facts and formulas, and helps to guide discovery learning through STEM.

REFERENCE

Partnership for 21st-Century Skills (P21) 2011. *Framework for 21st Century Learning*. Tucson, AZ: P21. http://www.p21.org/overview/skills-framework

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