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Effective Choices and Practices: Knowledgeable and Experienced Teachers' Uses of Manipulatives to Teach Mathematics

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This study was a comprehensive examination of the use of mathematics manipulative materials by a group of K-8 teachers identified as knowledgeable and experienced mathematics manipulative users. Because the teachers in this study were knowledgeable and experienced, the inquiry focused what mathematics materials the teachers chose to use. Analysis of over 500 lesson summaries from 116 K-8 teachers indicated that there were three common mathematics materials used by K-8 teachers who are knowledgeable and experienced manipulative users (dice, pattern blocks and snap cubes). The results support previous research showing that frequency of manipulative use declines from grades K through 8. In addition, the results revealed that the variety of manipulative used by teachers in this project also decreased across grades K through 8. An additional difference among the grade-specific groups was the way that teachers used the mathematics materials in their lessons, with Grades K-2 and 3-4 teachers using the materials in their lessons to develop an understanding of specific mathematical concepts, while Grades 5-6 and 7-8 teachers used the materials to engage students in open-ended investigations. These results demonstrate that K-8 teachers who are knowledgeable and experienced use the materials for mathematically rigorous content in their lessons.

Key words: Manipulative, mathematics, teachers, Grades K-8, instructional materials, virtual manipulative.

INTRODUCTION

In the recent climate of mathematics reform (e.g., Common Core State Standards), many important aspects of instruction have been considered in national and international studies. The Trends in International Mathematics and Science Study (TIMSS) used videotaped lessons to examine instruction in different countries, documenting aspects of teaching such as the materials used in the lessons [1,2]. The 1993 National Survey of Science and Mathematics Education [3] examined trends in mathematics education, including the use of mathematics materials in lessons. The "Inside the Classroom'' study conducted systematic observations of mathematics lessons, rating the quality of mathematics teaching on a variety of indicators including the manipulative used during the lessons [4]. A common theme among these studies is the role of mathematics materials for instruction.

Although numerous studies report what mathematics materials all teachers use for instruction [3,4], researchers actually know very little about what mathematics materials knowledgeable and experienced teachers use during instruction. Research reports that the most common mathematics materials used during instruction are manipulatives. This study was designed to take a fresh look at manipulative use. This perspective focuses on the use of manipulatives and other mathematics materials by Kindergarten through Grade 8 (K-8) teachers identified as knowledgeable and experienced mathematics manipulative

users. Knowledgeable and experienced mathematics manipulatives users were defined as teachers who had (a) training in the use of manipulatives, (b) access to the manipulatives, and (c) a self-identified interest in and intent for using the manipulatives during mathematics instruction. Because the teachers in this study were knowledgeable and experienced, our inquiry did not focus on whether or not the teachers would use the manipulative. Therefore, the research question focused on WHAT manipulative teachers would select, in terms of (a) teachers' choice of mathematical content, (b) grade-level appropriateness of mathematics content selected, and (c) pedagogy during lessons.

Using Mathematics Materials to Teach Mathematics

Constructivist theorists believe that learning is mediated by tools, and therefore, the "tool changes the form, structure, and character of the activity" [5]. Hiebert *et al.* [6] discuss this idea in their book, Making Sense. "Different tools are different forms of representation, and each conveys a somewhat different message, and each emphasizes somewhat different features of the idea" [p. 58]. Therefore, the mathematics tools students use can change the way they think about a concept.

Tools are important components of representational systems. Teachers have historically used multiple representations to teach mathematics. Representations commonly used in school mathematics include physical or concrete representations (e.g., manipulative and geometric models), visual or pictorial representations (e.g., pictures, graphs, and diagrams), symbolic or abstract representations (e.g., letters, operation signs, and numerals), and virtual manipulatives (defined as "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" [7 p. 373]. Representational fluency [8] and representational systems play an important role in students' understanding of concepts [9].

Research on Manipulatives and Other Mathematics Materials Used as Representations

The research on teachers' uses of manipulatives for mathematics instruction spans four decades [10-19]. Much of this research supports the use of manipulatives in the development of efficient mathematical strategies [2]. However, the way in which manipulatives are used is subject to the interpretations of teachers and students in the environments in which they teach and learn. For example, studies on manipulatives have shown that student achievement levels are related to teachers' experience in using the materials [15,16,19]. Meira [20] argues that different tools have varying degrees of transparency, or access, mediated by users' participation in activities with those tools. Therefore, teacher decisions about manipulative use are critical in students using tools effectively and constructing meaning.

Prior research demonstrates that frequency of manipulative use by teachers decreases as the grade level increases [3,19,21,22]. Other researchers have examined how manipulatives are used during classroom instruction. Moyer [11] found that the most common use of manipulatives by teachers was for the exploration of geometry concepts (35 percent of lessons) and to play a game (30 percent of lessons). Other studies have focused on how manipulatives influence students' attitudes [16], and what students do with the manipulatives when teachers provide students with free access to the materials [12].

Other mathematics materials, such as calculators and computers, have been shown to be effective for mathematics instruction. A long history of research on calculators has shown them to be effective tools for instruction [23-27]. More recent studies of computer use as a tool for mathematics instruction have also produced favorable results. Virtual manipulatives [7], which are dynamic objects which can be found at the National Library of Virtual Manipulatives (http://nlvm.usu.edu/), are frequently used in classrooms. A recent meta-analysis on the effects of virtual manipulatives as an instructional treatment showed moderate effects in favor of the virtual manipulatives when compared with other instructional treatments [28].

The Present Study

This study provides a fresh perspective on the use of manipulatives and other mathematics materials by examining K-8 teachers identified as knowledgeable and experienced mathematics manipulative users. The teachers in the present study are defined as knowledgeable and experienced mathematics manipulative users because they had (a) training in the use of manipulatives, (b) access to the manipulatives in their schools and classrooms, and (c) a self-identified interest in using the manipulatives for mathematics instruction. Researchers knew that the teachers had experience using manipulatives and that they would use manipulatives in their teaching. For this reason, our research questions focused on WHAT manipulatives teachers selected. The following questions guided the analysis. When knowledgeable and experienced mathematics manipulative users teach mathematics; (a) what manipulatives and other materials do teachers select for mathematics instruction? What manipulatives are used in terms of: (i) teachers' choices of mathematics content? (ii) the grade-level appropriateness of the mathematics content selected? and (iii) pedagogy during the mathematics lesson? By using a comprehensive sample of over 100 teachers, from kindergarten through Grade 8, our goal was to reveal patterns in the teachers' choices.

METHODS

This study employed qualitative methods for the analysis of surveys and secondary source documents [29]. To identify the teachers as knowledgeable and experienced mathematics manipulative users, the researchers gathered background information using teacher surveys. To answer the research questions, researchers analyzed written summaries of teacher lessons following instruction. The lesson summaries served as a way to identify teachers' own descriptions of the materials they selected and how they used those materials during mathematics instruction [30].

Participants

Participants were 116 kindergartens through eighth-grade (K-8) teachers from one large school system in a metropolitan area (over 40,000 students). Over two academic years, the 116 teachers were participants in mathematics professional development groups that volunteered for the study, with two groups each in four grade-specific groups (K-2, 3-4, 5-6, and 7-8). The teachers participated in eight different teacher professional development institutes that started during the summer and concluded during the spring of the following year. The teacher participants were 93% White, 6% Black, and 1% Asian. Fifty-three of the 116 teachers (46%) held masters degrees. Almost all of the K-8 teacher participants were female (108 female, 8 male). The level of teaching experience in the group ranged from 1 to 32 years (mean = 12.3 yrs; mode = 8 yrs; median = 9 yrs). Participants were given the option of earning three graduate credits from the local university for their participation. A Teacher Practice Survey was used to gather background information about the teachers. The 116 teachers all indicated a self-identified interest in and experience with using manipulatives for mathematics instruction.

Procedures

The eight groups of teacher participants attended one-week summer mathematics institutes (40 hours) followed by four formal meetings during the academic year (8 hours). The teachers participated in the summer institutes in grade-specific groups (K-2, 3-4, 5-6, and 7-8) facilitated by four different instructors during two different summers and different academic years. The purpose of the institutes was professional development in mathematics teaching through the use of mathematical problem-solving experiences. Manipulatives and virtual manipulatives were two main resources used daily during the summer institutes. The teachers received a variety of materials (including manipulatives and teacher resource books) for their classrooms. These materials were selected by the four instructors and varied depending on the grade-specific institute that the teacher attended (Table 1). Along with these materials, instructors used additional manipulatives from their own personal resources.

Data Sources and Analysis

The primary sources of data in this study were written lesson summaries and surveys completed by the teachers.

Written lesson summaries. During the academic year, teachers developed, taught, and wrote lesson summaries for five mathematics lessons that used manipulatives or other mathematics materials. The lesson summaries were prepared by teachers following instruction and were a compilation of the teacher's plan for the lesson, a list of the elements included in the lesson when it was taught in the teacher's classroom, and a description of what the teacher and students actually did during the lesson. Standard information including, grade level, objectives, materials, handouts, procedures, and assessment, was required for each lesson summary. This descriptive information provided evidence of what materials teachers selected and used. Throughout the academic school year, teachers submitted electronic copies of the lesson summaries to their instructors.

Electronic copies of the lesson summaries were compiled on CDs. Five trained reviewers analyzed the lesson summaries. The reviewers were mathematics specialists at the grade levels of the lessons with classroom teaching experience. Reviewers were trained in methods of data analysis. Researchers conducted several different analyses on the lesson summaries. In the first analysis, researchers identified all of the materials teachers selected by grouping the materials into four major categories: commercial manipulatives, measuring tools, technology, and other.

The second analysis identified: (a) teachers' choices of mathematics content? (b) the grade-level appropriateness of the mathematics content selected? and, (c) pedagogy during the mathematics lesson? First, researchers examined the relationship between the mathematics content in the lessons and the NCTM content standards [31]. Next, researchers examined the grade-level appropriateness of the lessons when manipulatives and other materials were used (examining content, objectives, and grade level with the state's mathematics standards). Researchers categorized each lesson as below, at, or above grade level based on these comparisons. The impetus for this analysis was to determine whether or not instruction represented appropriate grade-level content when teachers used mathematics materials.

The final part of the analysis employed a categorical system [32] to determine teachers' pedagogy when they used the mathematics materials during the lessons. Researchers identified seven categories describing

Table 1. Materials supplied to teachers by grade-specific groups.

K-2 Materials		3-4 Materials	
Math By All Means – Place Value	Overhead Hundred Number Boards	Overhead Counters	Overhead Fraction and Decimal Grids
Math By All Means – Geometry	Double Six Dominoes	Overhead Fraction Circles	Student Individual Clocks
Math By All Means – Probability	Overhead Version - Dominoes	Overhead Pattern Blocks	Customary Weights
Plastic Pattern Blocks (.5cm)	The Educator Overhead Calculator	Overhead Tangrams	Hexagon Metric Masses
Dual Clock Face Rubber Stamp	Wooden Geometric Solids	Overhead Attribute Blocks	18 Piece Liquid Measure Kit
Color Tiles	300 Baby Bear Counters	Overhead Clock Dials	Graphing Mat A
5 Transparent Spinners	Coin Head Stamps	Overhead Geoboard	Fraction Bar Game
Tangrams Class Set	Coin Tails Stamps	Overhead Spinners	Nimble with Numbers 3-4
Overhead Pattern Blocks	Tangramables	Overhead Coins	Getting Smarter Everyday 3-5
Overhead Base 10 Blocks	Pattern Animals	Overhead Bills	Navigating Through Geometry 3-5
18 Piece Liquid Measure	100 Color Cubes	Overhead Base 10 Blocks	Navigating Through Algebra 3-5
		Overhead Color Tiles	Overhead Rainbow Fraction Tiles
5-6 Materials		**7-8 Materials**	
Big Base 10 Kit Class Set		Easyshapes Pattern blocks (1cm thick)	Equation Dominoes
Geometry Stamp Kit		Overhead Algebra Tiles Set	Algebra Domino Links
Reflect View		Easyshapes Operation Dice (set of 6)	Lessons for Algebraic Thinking
Safe T Compass Class Set		Basic Clear Ruler (12in./30cm)	Overhead Pattern Blocks
Power Solids Class Set		Patty Paper Geometry Set	Multilink Cubes (set of 500)
Power Polygons Class Set		Angle Study Dominoes	The Middle School Mathematician
Tangram Class Set		Power Solids	Thought Provokers
St Thermometers Class Set		Investigating With Power Solids	More Thought Provokers
Overhead Base 10 Blocks		Algebra Tiles Workbook	Pre-Algebra Bingo
Geoboard Class Set			

pedagogy using a constant comparative method [30]. Teachers used the mathematics materials in the following ways: (a) investigate, (b) understand, (c) introduce, (d) game, (e) aide, (f) model, and (g) extend. Investigate was used to code lessons in which students engaged in open-ended investigations or problem-solving activities using the mathematics materials. In these lessons, teachers allowed students to explore the mathematics without leading them through the development of the mathematics concept. Understand was used to code lessons in which students used the mathematics materials to develop an understanding of specific mathematical concepts, often through a step-by-step or directed process, and then reinforce those concepts through independent practice. Introduce was used to code lessons in which the mathematics materials were used primarily to introduce a new concept. The students used the mathematics materials with teacher guidance during the introduction, but did not use it during the remainder of the lesson. In these lessons, teachers led students to focus on specific mathematics content. A lesson was coded as a game if students primarily used the mathematics materials to play a game which was often described as a fun activity where the mathematical purpose of the lesson may or may not be made explicit to students. Aide was used to code lessons in which the mathematics materials were used primarily for remediation or to assist students who were having difficulty, rather

than to assist the whole class. Model was used to code lessons in which the teacher demonstrated or modeled a concept with the mathematics materials, but the students did not use the materials themselves during the lesson. Extend was used to code lessons in which the mathematics materials were used primarily to extend a concept for students who were achieving above grade level, rather than for use with the whole class.

Teacher practice surveys. Teachers completed a Teacher Practice Survey twice that served as a self-report of their use of and familiarity with the mathematics materials. The surveys also recorded demographic information. Teachers completed the surveys during regularly scheduled group meetings; therefore, both survey administrations had very high (85-100%) rates of return for all groups. All of the 116 teachers completed the survey during the first administration (summer), and 104 teachers completed the survey during the second administration (spring of the academic year). The first survey administration asked teachers to report on the mathematics materials they had used the academic year prior to the summer professional development. The second survey administration asked teachers to report on the mathematics materials teachers used during the current academic year. Researchers used the responses to: (a) provide background information on the teachers to confirm their prior use of mathematics materials for instruction, (b) identify specific mathematics materials the teachers used, and, (c) examine patterns among the grade-specific groups. Researchers summarized the survey responses as averages and percentages.

RESULTS

The results are organized in two major sections: (a) background information on the teachers and their prior use of manipulatives and (b) what manipulatives and other materials the teachers used, in terms of teachers' choices of mathematics content, the grade-level appropriateness of the mathematics content selected, and their pedagogy during the lessons.

Background on Teachers as Knowledgeable and Experienced Manipulative Users

To categorize the teachers as "knowledgeable and experienced manipulative users," the researchers used teacher survey data to identify the types of manipulatives and other materials teachers used prior to and during the study and their frequency of use.

Background on teachers' use of manipulatives. The Teacher Practice Surveys included a list of manipulatives with the question; Circle the mathematics manipulatives you have USED during the academic year (summer and spring surveys). Table 2 shows the percentages of teachers in each grade-specific group reporting the use of manipulatives listed on the survey. Teachers reported using a variety of different manipulatives prior to and during the study including four common manipulatives: dice, pattern blocks, snap cubes and color tiles. Grades K-2 and 3-4 teachers reported using a greater variety than Grades 5-6 and 7-8 teachers. The highest manipulative use reported on the spring surveys for K-2 teachers were dice, dominoes, pattern blocks, snap cubes and color tiles; Grades 3-4 were dice, spinners, base-10 blocks, color tiles, plastic coins/bills, and transparent counters; Grades 5-6 were dice, base-10 blocks, geoboards, pattern blocks, and tangrams; and Grades 7-8 were snap cubes, pattern blocks, geometric solids, and color tiles.

Background on teachers' use of measurement tools. The ruler was the most frequently used measurement tool in all four grade-specific groups, with 93% of all teachers reporting that they used rulers, followed by the use of balance scales and measuring tapes, 58% and 52% respectively (Table 3). The measurement tools reported most often by K-2 teachers were balance scales and clock materials. Teachers in Grades 3-4 reported rulers, balance scales, and clock materials with the highest frequency on their surveys. In Grades 5-6, rulers, protractors, and compasses were reported most often on surveys. Every 7-8 teacher reported ruler use.

Background on teachers' use of technology. Teachers reported using calculators as their most frequent technology prior to and during the study (Table 4). Other common technology reported by the teachers included CCC and virtual manipulatives. (CCC is a technology learning environment that individualizes instruction for students at their learning level by adjusting the presentation of procedural content.) Teachers' increased reporting on the use of virtual manipulatives in the spring administration of the survey was not surprising, because teachers were introduced to uses for virtual manipulatives by the instructors during the summer institutes.

Background on frequency of manipulative use. One item on the Teacher Practice Survey asked; How many days per week do you use manipulatives? On 104 matched surveys from summer and spring survey administrations, 14 were removed because: the respondent did not teach mathematics (1), question was blank (4), and respondent taught mathematics less than five days per week (9). On the 90 matched surveys that were left, teachers reported that they had used manipulatives 2.56 days per week (M = 2.56, SD = 1.59) the previous school year. Teachers reported that they were currently using manipulatives 3 days per week (M = 2.92, SD = 1.55). Grades K-2 teachers reported using manipulatives with the greatest frequency (pre, M = 3.63, SD = 1.39; post, M = 3.93, SD = 1.04), followed by Grades 3-4 (pre, M =

Table 2. Teachers' Reported Use of Manipulatives by Grade-Specific Groups Based on Surveys.

	Grade-Specific Groups									
	K-2 (N = 32)		3-4 (N = 24)		5-6 (N = 23)		7-8 (N = 25)		All (N =104)	
Manipulatives	**Su**	**Sp**	**Su**	**Sp**	**Su**	**Sp**	**Su**	**Sp**	**Su**	**Sp**
Algebra Dominoes	0	3	0	0	0	0	4	40	1	11
Algebra Tiles	3	9	4	8	9	4	44	52	14	18
Attribute Blocks	53	69	33	42	13	9	0	16	27	37
Base-10 Blocks	69	66	83	88	52	78	12	8	55	60
Color Tiles	63	91	54	88	22	48	16	60	40	73
Dice	75	97	83	96	83	96	68	52	77	86
Dominoes	66	94	29	21	17	13	0	32	31	44
Fraction Materials	38	41	71	67	61	61	20	40	46	51
Geoboards	69	63	42	33	52	70	8	20	44	47
Geometric Solids	75	78	75	46	48	48	56	64	64	61
Graphing Mats	59	44	25	71	30	9	12	16	34	36
Hundred Boards	53	66	71	83	30	22	4	8	40	46
Pattern Blocks	94	94	71	75	30	65	4	76	53	79
Plastic Coins, Bills	94	75	83	88	35	22	16	8	60	50
Snap Cubes	94	94	63	71	26	39	8	80	51	73
Spinners	59	78	71	96	57	52	40	40	57	67
Tangrams	72	84	71	58	57	65	20	44	56	64
Transparent Counters	41	53	54	88	30	17	12	24	35	46
Two-Color Counters	78	84	50	67	35	43	32	32	51	59

Note: Because groups contain different *N*s, data are presented as percents for comparison purposes. Su = survey administered in the summer mathematics institute; Sp = survey administered in the spring of the academic year.

Table 3. Teachers' Reported Use of Measurement Tools by Grade-Specific Groups Based on Surveys.

	Grade-Specific Groups									
	K-2 (*N* = 32)		3-4 (*N* = 24)		5-6 (*N* = 23)		7-8 (*N* = 25)		All (*N* = 104)	
Measurement Tools	Su	Sp	Su	Sp	Su	Sp	Su	Sp	Su	Sp
Balance Scales	84	75	54	83	43	48	20	20	53	58
Clock Materials	97	81	92	83	35	17	12	4	62	49
Compasses	13	22	17	21	57	61	60	60	35	39
Liquid Measure Kits	41	47	50	75	39	26	12	4	36	38
Measuring tapes	56	53	33	46	48	57	48	52	47	52
Protractors	3	0	13	4	74	65	80	68	39	32
Rulers	94	84	96	96	100	96	100	100	97	93
Weights	59	47	63	79	52	39	12	8	47	43

Note: Because groups contain different *N*s, data are presented as percents for comparison purposes. Su = survey administered in the summer mathematics institute; Sp = survey administered in the spring of the academic year.

2.81, SD = 1.25; post, M = 3.48, SD = 1.03), Grades 5-6 (pre, M = 2.05, SD = 1.23; post, M = 2.50, SD = 1.67), and Grades 7-8 teachers who reported the least frequency (pre, M = 1.45, SD = 1.53; post, M = 1.54, SD = 1.22). This trend showing declining frequency of manipulative use across the grade levels is consistent with prior research.

Summary of teacher background information. As the surveys show, teachers in the grade-specific groups were consistent in reporting their frequency of manipulative

Table 4. Teachers' Reported Use of Technology by Grade-Specific Groups Based on Surveys.

	Grade-Specific Groups									
	K-2 (*N* = 32)		3-4 (*N* = 24)		5-6 (*N* = 23)		7-8 (*N* = 25)		All (*N* = 104)	
Technology	**Su**	**Sp**	**Su**	**Sp**	**Su**	**Sp**	**Su**	**Sp**	**Su**	**Sp**
Calculators	56	69	83	79	96	96	100	100	82	85
CCC	34	38	96	83	48	43	0	0	43	40
Geometer's Sketchpad	0	0	0	0	4	4	12	48	4	13
Graphing Programs	47	47	33	50	17	9	12	24	29	34
Math Processor	38	47	13	50	4	0	0	0	15	26
Spreadsheets	34	28	33	21	52	26	28	36	37	28
Tesselmania	3	0	8	4	17	17	12	16	10	9
Virtual Manipulatives	9	75	25	83	9	83	8	56	13	74

Note: Because groups contain different Ns, data are presented as percents for comparison purposes. Su = survey administered in the summer mathematics institute; Sp = survey administered in the spring of the academic year.

use. The increases can be accounted for by their training during the project. The surveys demonstrate that teachers had; (a) prior experience in the use of mathematics materials (based on their self reports); (b) training in the use of mathematics materials (including 48 hours of professional development during the project); (c) access to a variety of mathematics materials (which they received during the project); and, (d) a self-identified interest in the use of mathematics materials (based on voluntary participation in the project and increased use of manipulatives during the project). These characteristics defined the teachers as knowledgeable and experienced manipulative users.

What Manipulatives and Other Materials Do Knowledgeable and Experienced Manipulative Users Select for Mathematics Instruction?

The primary research question asked: What manipulatives and other materials do teachers select for mathematics instruction? To answer this question, we reviewed mathematics lesson summaries prepared by teachers. Teachers were asked to design five lessons, which should have resulted in a total of 580 lesson summaries (116 teachers x 5 lessons = 580 lessons). The summaries submitted were reviewed for completeness (all elements included in the submission), repetition (lesson summaries that were duplicates), and clarity (enough information to identify content and pedagogy). Lessons that did not meet these criteria were removed from the analysis. Some teachers wrote less than the required five lesson summaries, resulting in only 537 lesson summaries. The lessons submitted represented each grade-specific group (Figure 1). The following sections report teachers' selections of mathematics materials in four major categories: commercial manipulatives, measuring tools, technology, and other materials.

What commercial manipulatives do teachers select for their lessons? The commercial manipulatives teachers used during instruction are presented in Table 5 using percentages for ease of comparison among the groups. The manipulatives used most frequently by all teachers were dice, pattern blocks, and snap cubes. K-2 teachers most commonly used teddy bear counters, pattern blocks, dice, snap cubes, and dominoes. Grades 3-4 teachers most commonly used dice, transparent counters, base-10 blocks, and plastic coins/bills. Grades 5-6 teachers used power polygons, geoboards, base-10 blocks, geometric solids, and dice. Grades 7-8 teachers most commonly used pattern blocks, snap cubes, algebra tiles, and geometric solids. Eight commercial manipulatives were used by all four grade-specific groups (pattern blocks, dice, snap cubes, geometric solids, geoboards, transparent counters, tangrams, and fraction materials). To determine variety of use, we examined manipulatives used by teachers in at least 1% of the lessons at each grade level. Using this definition of variety, Grades K-2 teachers used a greater variety of manipulatives in their lessons (18 different manipulatives), with a decrease in variety for each of the other grade-specific groups (Grades 3-4 = 16; Grades 5-6 = 13; Grades 7-8 = 10).

What measurement tools do teachers select for their lessons? Teachers reported the ruler as the most frequently used measurement tool (6.9% of lessons). Rulers were reported almost three and a half times as

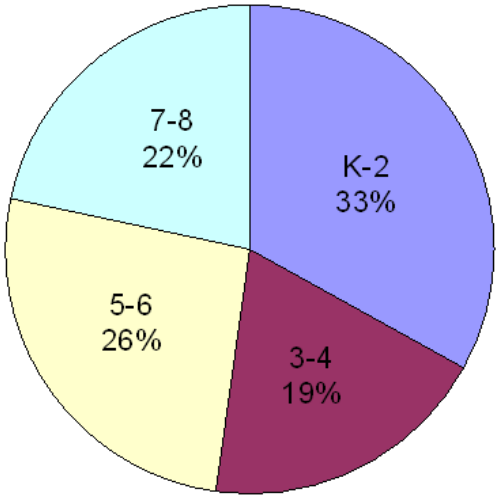

Figure 1. Distribution of teachers' lesson summaries by grade-specific groups.

Table 5. Teacher Use of Manipulatives by Grade-Specific Groups Based on Lesson Summaries.

Mathematics Materials	Grade-Specific Groups				
	K-2 (*N* = 177)	3-4 (*N* = 104)	5-6 (*N* = 140)	7-8 (*N* = 116)	All (*N* = 537)
Algebra Dominoes	0.0	0.0	0.0	1.7	0.4
Algebra Tiles	0.0	0.0	0.0	4.3	0.9
Attribute Blocks	1.7	0.0	1.4	0.0	0.9
Base-10 Blocks	2.3	9.6	12.1	0.0	5.8
Color Tiles	4.5	2.9	0.7	0.0	2.2
Dice	10.2	14.4	6.4	2.6	8.4
Dominoes	8.5	0.0	0.0	0.9	3.0
Fraction Materials	0.6	4.8	4.3	0.9	2.4
Geoboards	0.6	1.0	13.6	0.9	4.1
Geometric Shapes	1.7	0.0	0.0	0.0	0.6
Geometric Solids	4.0	1.9	7.9	6.0	5.0
Graphing Mats	1.1	5.8	0.0	0.0	1.5
Hundred Boards	2.8	2.9	0.0	0.0	1.5
Mirrors	0.0	1.0	2.9	0.9	1.1
Pattern Blocks	11.3	4.8	0.7	14.7	8.0
Plastic Coins, Bills	7.3	8.7	0.0	1.7	4.5
Power Polygons	0.0	0.0	18.6	1.7	5.2
Snap Cubes	9.0	1.0	2.9	13.8	6.9
Spinners	2.8	2.9	1.4	0.0	1.9
Tangrams	2.8	1.0	3.6	0.9	2.2
Teddy Bear Counters	11.9	0.0	0.0	0.0	3.9
Transparent Counters	3.4	13.5	1.4	1.7	4.5
Two-Color Counters	3.4	0.0	0.0	0.0	1.1
Other	2.8	1.9	2.9	2.6	2.6

Note: Because groups contain different Ns, data are presented as percents for comparison purposes.

Table 6. Teacher Use of Measurement Tools by Grade-Specific Groups Based on Lesson Summaries.

Measurement Tools	Grade-Specific Groups				
	K-2 (*N* = 177)	3-4 (*N* = 104)	5-6 (*N* = 140)	7-8 (*N* = 116)	All (*N* = 537)
Balance Scales	2.8	1.9	2.1	0.9	2.0
Clock Materials	1.7	3.8	0.0	0.0	1.3
Compasses	0.0	0.0	0.7	1.7	0.6
Liquid Measure Kits	1.1	2.9	0.7	0.9	1.3
Measuring Tapes	0.6	1.9	0.0	0.9	0.7
Protractors	0.0	0.0	2.9	5.2	1.9
Rulers	0.6	1.9	8.6	19.0	6.9
Scales	0.6	1.9	0.0	0.0	0.6
Weights	0.0	1.9	0.0	0.0	0.4

Note: Because groups contain different Ns, data are presented as percents for comparison purposes.

Table 7. Teacher Use of Technology by Grade-Specific Groups Based on Lesson Summaries.

Technology	Grade-Specific Groups				
	K-2 (*N* = 177)	3-4 (*N* = 104)	5-6 (*N* = 140)	7-8 (*N* = 116)	All (*N* = 537)
Calculators	3.4	1.9	8.6	13.8	6.7
CCC	1.1	0.0	0.0	0.0	0.4
Geometer's Sketchpad	0.0	0.0	2.1	12.1	3.2
Graphing Programs	5.1	2.9	0.0	0.9	2.4
Kid Pix	10.2	2.9	0.0	0.0	3.9
Math Processor	2.3	6.7	0.0	0.0	2.0
Spreadsheets	0.0	0.0	0.0	0.0	0.0
Tesselmania	0.0	0.0	0.0	0.0	0.0
Virtual Manipulatives	15.3	14.4	21.4	19.8	17.7
Other	2.3	4.8	2.1	1.7	2.6

Note: Because groups contain different Ns, data are presented as percents for comparison purposes.

often as the next reported measurement tools, which were balance scales (2.0% of lessons) and protractors (1.9% of lessons) (Table 6). The measurement tools used most often in K-2 lessons were balance scales, clock materials, and liquid measure kits; in Grades 3-4 were clock materials and liquid measure kits; in Grade 5-6 were rulers, protractors, and balance scales; and in Grade 7-8 were rulers. Nineteen percent of the Grade 7-8 lessons included rulers, which was more than double the next highest percentage of any measurement tool reported at any grade level.

What technology do teachers select for their lessons? The two most common technologies used in teachers' lessons were virtual manipulatives and calculators (Table 7). Almost one-fifth (17.7%) of all lessons included virtual manipulatives, followed by 6.7% that used calculators. Calculator use was greater at Grades 5-6 and 7-8 (8.6% and 13.8%, respectively) than Grades K-2 and 3-4 (3.4% and 1.9%, respectively). Teachers' high use of virtual manipulatives was caused by the instructors who asked the teachers to use them in one lesson.

What other materials do teachers select? Teachers identified the use of other materials in their lessons that could not be categorized as commercial manipulatives, measurement tools, or technology. These other materials included craft items, paper items, toothpicks, and candy. Almost 15% of the K-2 mathematics lessons included food.

Descriptions of Manipulative Use During Instruction

What mathematics content do teachers select when they use mathematics materials? The second question

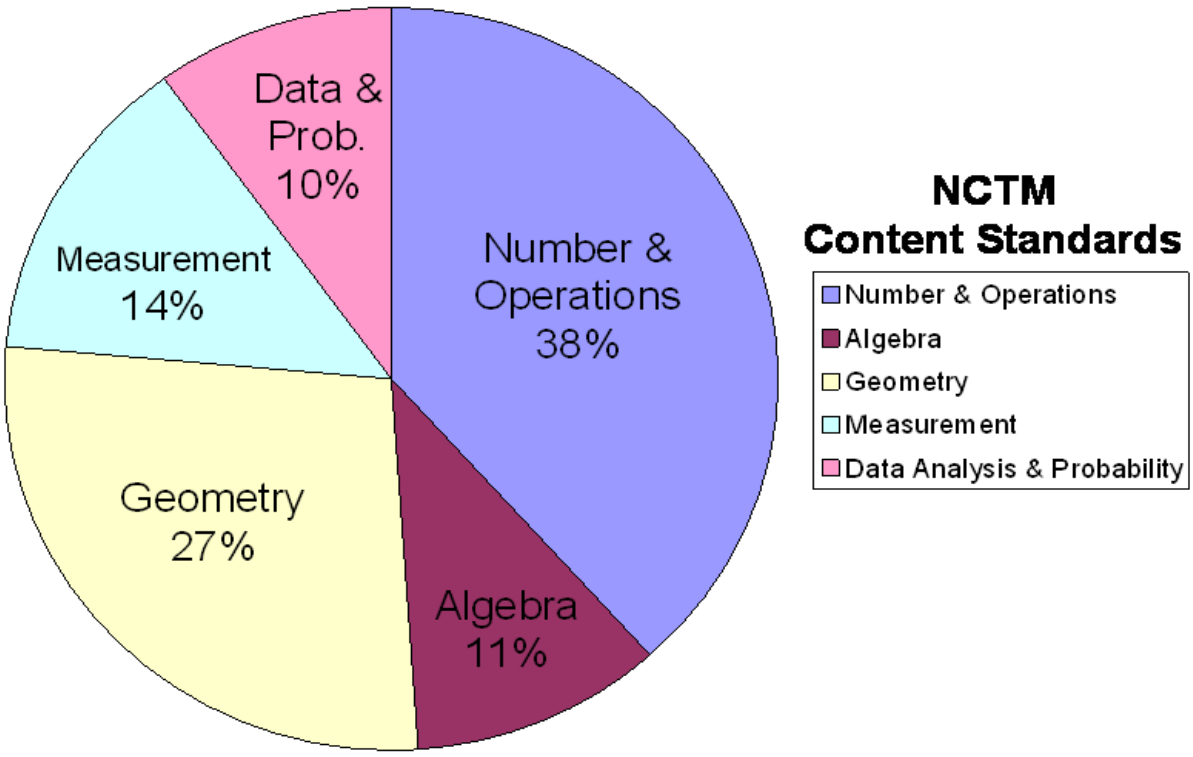

Figure 2. Distribution of content in the lesson summaries based on the NCTM Standards.

asked: What manipulatives are used in terms of: (i) teachers' choices of mathematics content? (ii) the grade-level appropriateness of the mathematics content selected? and (iii) pedagogy during the mathematics lesson?

Content in relation to the NCTM standards. The researchers categorized the mathematics content of each lesson summary according to the NCTM content standards [31]. Lesson summaries that focused on more than one standard had the standard weighted in the analysis so that each lesson summary maintained a total weight of one. For example, if a lesson focused on both geometry and measurement, it was coded as .5 for geometry and .5 for measurement resulting in a total weight of 1.0 for the lesson summary. Figure 2 shows the mathematics content teachers selected when they used mathematics materials. Teachers selected the Number and Operations (38%) and Geometry (27%) content standards most frequently when they used mathematics materials.

In Figure 3, we compare teachers' content selections with the content recommendations in the NCTM standards [31]. Figure 3 indicates, the Number and Operations standard constituted the majority of lessons in three of the four groups (K-2, 3-4, and 5-6). In the remaining group (7-8), almost half of the summaries focused on Geometry (49%), and less than a quarter focused on Number and Operations (24%) and Algebra (19%). The Geometry standard was also prominent in groups 5-6 (31%) and K-2 (20%). These content selections show that teachers' emphasis on content follows the NCTM recommendations across grade bands [31]. Teachers' selections and the NCTM Standards show the Number and Operations, Measurement, and Data Analysis and Probability standards generally decreasing, and the Algebra and Geometry standards generally increasing in emphasis from Grades K through 8. In other words, when these teachers used mathematics materials, the content of their lessons was consistent with NCTM's recommendations for the emphasis of content across grades K-8.

Specific content topics selected. Next the researchers categorized specific topics within each NCTM standard to identify the topics teachers taught when they used mathematics materials. Again, we weighted topics in this analysis when more than one topic was the focus of a lesson. Researchers collapsed the topics into manageable categories for reporting purposes. For example, researchers collapsed the content topics of ordinal numbers, even/odd numbers, and prime/composite numbers into one category called categories and properties of numbers. Figure 4 shows the distribution mathematics topics.

In the Number and Operations standard, teachers used

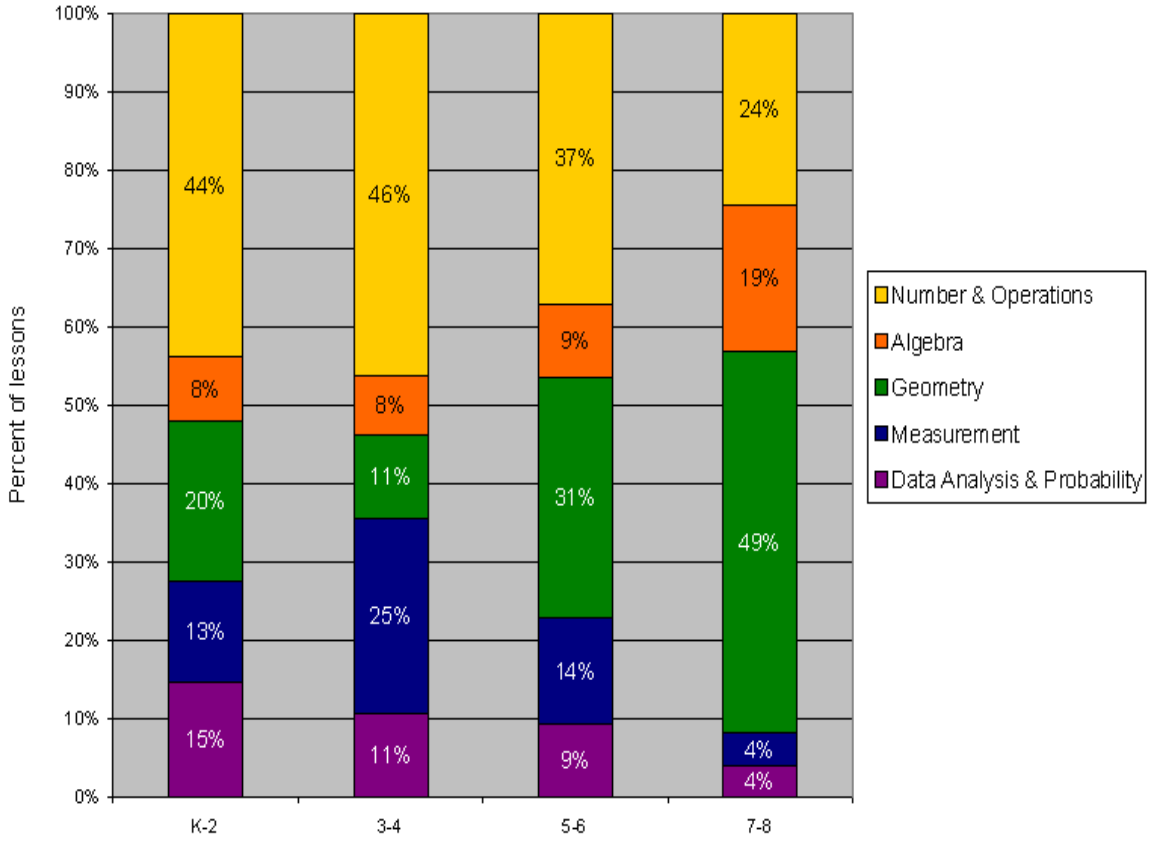

Figure 3. Distribution of content in the lesson summaries by grade-specific groups based on the NCTM Standards.

mathematics materials to focus on rational numbers and addition/subtraction. In the Algebra standard, teachers focused on patterns in more than half of the lessons, followed by equations/inequalities. In the Geometry standard, teachers focused on plane shapes, transformations, solid shapes, and angles. In the Measurement standard, teachers focused instruction on money, area/perimeter, length, and volume. In the Data Analysis and Probability standard, two-thirds of the lessons focused on graphing.

What is the grade level appropriateness of the lesson when teachers use mathematics materials? Next, we examined the lessons to determine how grade-level appropriate the content was when teachers used mathematics materials. In this analysis, 111 (of 537) lesson summaries were removed because; 1) the teacher stated no grade level or more than one grade level, or 2) there was not enough information in the lesson summary to assign a state objective. The 426 lessons were analyzed by comparing teacher-identified objectives and grade levels to the state's mathematics standards. Algebra and Geometry were listed separately because these were taught as separate courses in middle schools. Table 8 indicates, 77% of the lessons were identified *at* grade level. About an equal number of lessons were identified below and *above* grade level, 12% and 11% respectively. The largest number of lessons categorized below grade level was in Grades 6, 7, and 8. Grades K and 1 contained the largest number of lessons categorized above grade level. Therefore, when these teachers taught lessons using mathematics materials, a majority of their lesson were at or above grade level in mathematical content.

What is the pedagogy in lessons where mathematics materials are used? In the final analysis, 537 lessons were examined to determine the pedagogy reported for the lesson when mathematics materials were used. Table 9 indicates that teachers used mathematics materials in a majority of the lessons to investigate mathematical ideas or to understand mathematical concepts (36% and 35%, respectively). A greater number of lessons were designed to develop understandings at Grades K-2 (44%) and 3-4 (37%). An example of a lesson designed to help first graders understand the concepts of addition and

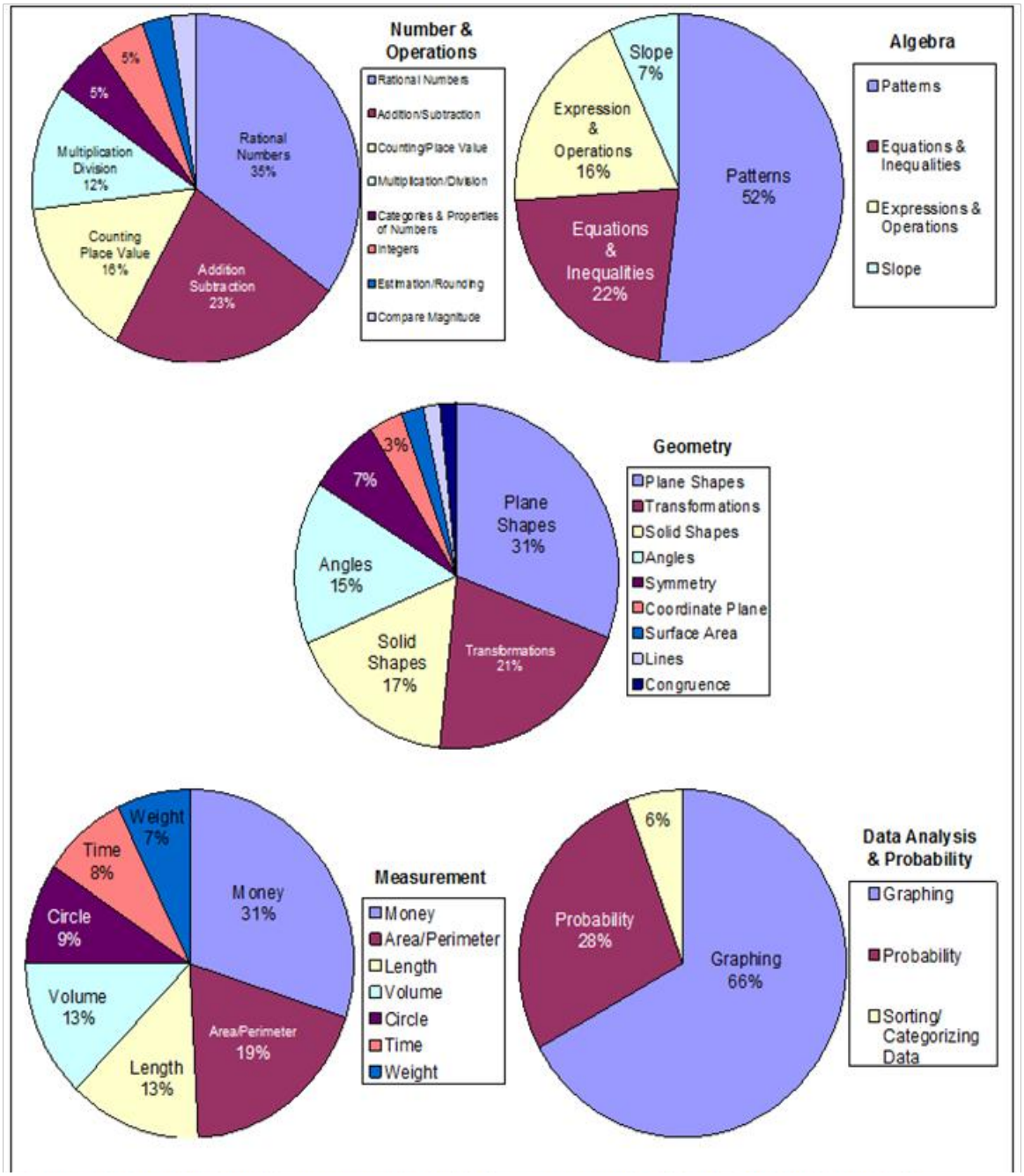

Figure 4. Distribution of content topics in the lessons organized by the NCTM Standards.

subtraction engaged students in creating fact families using teddy bear counters. After the teacher provided several examples to guide students' thinking, the students created their own stories using the bears and wrote addition and subtraction equations like those shown by the teacher. An example of a lesson developed to help third graders understand place value used dice and virtual base-ten blocks. During this lesson, students practiced their place value skills by rolling dice and created the largest possible number with the digits rolled. They modeled the number with the virtual base-ten blocks and wrote the number in standard form, expanded form, and words.

There were a greater number of lessons that included open-ended investigations at Grades 5-6 (43%) and 7-8 (47%). An example of a Grades 5-6 lesson designed to investigate properties of triangles included the use of geoboards. During this lesson students created as many triangles as they could on the geoboards, and then discussed and recorded similarities and differences of the triangles with classmates. An example of a grades 7-8 lesson designed to investigate angle measurements used

Table 8. Grade-Level Analysis of Lesson Summaries (N = 426).

Grade	*N* Lessons	Below	At	Above
K	32	0(0%)	22(69%)	10(31%)
1	68	1(1)	51(75)	16(24)
2	49	4(8)	39(80)	6(12)
3	79	7(9)	70(89)	2(3)
4	31	3(10)	24(77)	4(13)
5	46	7(15)	37(80)	2(4)
6	56	20(36)	32(57)	4(7)
7	19	4(21)	13(68)	2(11)
8	16	5(31)	11(69)	0(0)
Algebra	13	0(0)	13(100)	0(0)
Geometry	17	0(0)	17(100)	0(0)
All	426	51(12)	329(77)	46(11)

Table 9. Pedagogy when mathematics materials were used in the lessons.

	Grade-Specific Groups				
	K-2 (*N* = 177)	3-4 (*N* = 104)	5-6 (*N* = 140)	7-8 (*N* = 116)	All (*N* = 537)
Investigate	27	29	43	47	36
Understand	44	37	29	25	35
Intro	11	16	17	1	15
Game	16	13	7	7	11
Other (Aide, Model, Extend)	2	5	3	2	3

Note: Because groups contain different Ns, data are presented as percents for comparison purposes.

Geometer's Sketchpad. In this lesson, students explored and constructed angles by cutting parallel lines with transversals, followed by discussions that led to discovering relationships among the angle measures. Mathematics materials were used with less frequency to introduce concepts (15% of all lessons); for a game (11% of all lessons); or to aide, model, or extend concepts during the lessons (less than 5% of all lessons).

During the lessons, teachers sometimes used a mathematics material alone and other times used a combination of materials in the following ways in the lessons: commercial manipulatives only (34%), measurement tools only (5%), and technology only (17%), or they used multiple materials in the lesson (35%).

Limitations

In the present study, a portion of the use of commercial manipulatives in the lessons can be attributed to the distribution of those manipulatives during the project. The mathematics materials used most frequently by teachers in their lessons were often materials that were given to them. For example, Grades K-2 teachers were given teddy bear counters and used them in almost 12% of their lessons, and Grades 5-6 teachers were given power polygons and used them in almost 19% of their lessons. Teachers wrote lessons utilizing the virtual manipulatives because they were asked by the instructors to create at least one lesson utilizing this technology.

DISCUSSION

The purpose of this study was to examine the mathematics materials used by teachers identified as knowledgeable and experienced manipulative users. This allowed us to focus on what manipulatives knowledgeable and experienced teachers selected for their lessons in terms of content, grade level appropriateness, and pedagogy, and how the selection and use of mathematics materials varied across Grades K-8.

These results show some important insights into mathematics materials use by knowledgeable and experienced teachers. While some studies conclude that

teachers' uses of mathematics materials focus on "fun" or "games" [11], these results demonstrate that teachers who are knowledgeable and experienced will (a) use mathematics materials to teach standards-based content, (b) ensure that the mathematics content is grade appropriate, and (c) focus their pedagogy on understanding and investigating mathematical concepts. These teachers were not simply "having fun" or having their students "play" with the materials.

The Mathematics Materials Teachers Selected

The results show that there were similarities and differences among grade-specific groups in the mathematics materials teachers selected.

Similarities in materials selections. There were common mathematics materials used among the grade-specific groups. Most frequently used by all teachers were dice, pattern blocks, and snap cubes. In addition, all groups used eight common commercial manipulatives in their lessons (dice, pattern blocks, snap cubes, geometric solids, geoboards, transparent counters, tangrams, and fraction materials). In essence, these commercial manipulatives have the highest utility value across the groups. The ruler was used at every grade level and was the most frequently used measurement tool. Virtual manipulatives and calculators were the most common technology used across the groups.

Research on mathematics materials shows that the specific tools students use for learning mathematics influence the kinds of understandings students develop [33]. This has important implications for school district leaders when allocating budget resources for mathematics materials. The mathematics materials used with higher frequency and across multiple grade levels may be ones that have more utility for school-based mathematics, and therefore, are more economical for school systems to acquire. Examining these grade-specific results could assist school system planners in identifying the commercial manipulatives most utilized at different levels of instruction.

Differences in materials selections. Some commercial manipulatives were only used in one or two of the grade-specific groups (e.g., teddy bear counters, K-2; Power Polygons, 5-6 and 7-8; algebra tiles, 7-8). These commercial manipulatives may have been most appropriate for a specific topic at a limited grade level (i.e., algebra dominoes, that display equations and expressions, would be appropriate to limited grade levels).

In terms of measurement tools, Grades 7-8 teachers used rulers more frequently than teachers at other grade levels. While a small percentage of Grades 7-8 lessons focused on measurement concepts (4%), rulers were used in 19% of the Grades 7-8 lessons. Teachers may have integrated measurement concepts with other topics such as geometry or they used rulers for many other purposes in addition to teaching measurement.

Variety in materials selections. The results showed that there were differences among the grade-specific groups in terms of the variety of manipulatives teachers used. For example, K-2 teachers used 18 different commercial manipulatives, as compared with Grade 7-8 teachers who used 10 different commercial manipulatives. Gravemeijer [34] describes mathematical tools as "models for thinking." The mathematics materials the teacher selects can influence different kinds of understanding and can influence the way students think about the mathematical activity in which they are engaged [6]. Whether or not greater variety in manipulative use adds to or detracts from students' thinking and learning in lessons where manipulatives are used may be determined by this selectivity.

In examining the number of different manipulatives provided to teachers during the institutes, the Grade 5-6 group received the smallest variety of materials, and the Grade 3-4 group received more overhead materials for teacher use. Therefore, the variety of materials teachers received during the teacher institutes did not influence the results which showed that teachers in the lower grades used a greater variety of manipulatives than teachers in the upper grades. A factor other than the materials teachers received influenced the variety of manipulatives teachers used and the decline of that use across the grade-specific groups.

Teachers' Pedagogy When Mathematics Materials Were Used

The researchers' examination of how knowledgeable and experienced teachers used mathematics materials focused on (1) content, (2) grade level appropriateness, and (3) instructional pedagogy. Teachers used materials in lessons across all five content areas in each of the four grade-specific groups. The NCTM content standards analysis showed appropriate coverage of the content standards for each grade-specific group based on NCTM's recommendations. For example, there was a lesser emphasis on number and operations and a greater emphasis on algebra along the continuum of grades K through 8.

The examination of grade level appropriateness showed that a majority of the lessons taught by knowledgeable and experienced manipulative users were at or above grade level in terms of the mathematical content. The largest number of lessons that were categorized as below grade level appeared in Grades 6, 7, and 8. One possible explanation for this pattern is that students in the middle grades are frequently tracked into classes at different levels and teachers may be adapting content for lower achieving students. Another trend in

middle schools is to hire teachers with elementary education certification and less mathematics subject area preparation. Elementary-certified teachers are in essence, teaching out-of-field, and may not have the level of content preparation necessary to teach mathematics at the level required for Grades 6 through 8. The largest number of lessons that were categorized as above grade level appeared in Grades K and 1. One possible explanation for this pattern may be that the mathematics materials were used in lessons as conceptual supports which enabled teachers to extend children's learning beyond grade level requirements and provide younger students with access to mathematics at higher levels. Another possible explanation is that the standards in Grades K and 1 are at a low level and many children know the mathematics content of these grades before they come to school forcing the teachers in to address standards beyond their grade level to meet students' needs. These findings dispel notions that all teachers who are using manipulatives are using these materials to simplify or "dumb down" the mathematics for their students.

The results showing teachers' instructional strategies when mathematics materials were used indicated that most lessons focused on students understanding concepts or investigating mathematics, with a greater number of lessons focused on developing understandings in Grades K-2 and 3-4, and a greater number of lessons focused on open-ended investigations in Grade 5-6 and 7-8. With less frequency, the materials were used to introduce new concepts. Few lessons used materials to aide, model, or extend concepts. In contrast with previous findings on manipulative use, Moyer [11] reported that 30% of observed lessons used manipulatives in a game, whereas only 11% of the lessons in the present study did so. The difference between teachers who are knowledgeable and experienced with manipulatives and those who are not may explain the different study results.

The Influence of Professional Development

Research suggests that professional development designed around the use of curriculum materials results in the increased use of those materials in teachers' instructional practice [35,36]. The most successful implementation results have been reported when professional development is paired with the adoption of manipulatives for mathematics instruction [37,38]. In the present study this was true of the mathematics materials provided and used in the project; they appeared frequently in teachers' lessons. However, as teachers' surveys indicated, many of the materials were ones teachers had used prior to the project. Therefore, teachers began with an experiential base on which to build their new learnings. The high use of virtual manipulatives shows the influence that the professional development had on teachers' lessons and instructional planning during the year following the summer institutes. Virtual manipulatives were the most frequently used technology and the most frequently used of all mathematics materials at every grade level. While virtual manipulatives were a new tool for many of the teachers in the study (i.e., only 13% used virtual manipulatives prior to the project), teachers reported a 61 percentage point increase in their use during the project.

Conclusions

The results of this study show that there are some common mathematics materials used by K-8 teachers who are knowledgeable and experienced manipulative users. The results also support previous findings indicating that teachers' frequency of manipulative use declines from grades K through 8 [3,21,22]. The present study also revealed that the variety of manipulatives used by teachers in this project decreased across grades K through 8. This suggests that teachers in the early grades not only use manipulatives with greater frequency, but they may also use a greater variety of manipulatives. An additional difference among the grade-specific groups was the way that teachers used the mathematics materials in their lessons. Teachers in Grades K-2 and 3-4 used the mathematics materials to develop an understanding of specific mathematical concepts, often through a step-by-step or directed process, and then reinforce those concepts through independent practice. Teachers in Grades 5-6 and 7-8 used the mathematics materials to engage students in open-ended investigations or problem-solving activities which allowed students to explore the mathematics without leading them through the development of the mathematics concept. In contrast with previous research that implies that when teachers use manipulatives to teach mathematics their students are simply having fun [11], these results demonstrate that teachers who are knowledgeable and experienced manipulative users are selective in the use of mathematics materials, they use the materials effectively with appropriate mathematical content topics, and their lessons are designed with the mathematics materials to be grade-level appropriate.

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