Chapter 13
Ambitious Teaching: Designing Practice-Based Assignments for Integrating Virtual Manipulatives into Mathematics Lessons

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Abstract This chapter details a design-based research study that focused on developing effective approaches for pre-service teachers to integrate technology in the mathematics classroom. Using an iterative design cycle, the researcher developed three practice-based assignments during an elementary mathematics methods course that were designed to promote pre-service teachers' technology pedagogical content knowledge. These practice based assignments allowed the participants to (a) analyze effective technology tools for mathematics teaching and learning; (b) evaluate applets that supported a vertical learning progression on a specific mathematics concept; and (c) design, implement and reflect on a mathematics lesson where technology amplified the mathematics teaching and learning. By creating pre-service teachers' own practical image of practice after implementing the technology integrated lessons in the field, pre-service teachers gained a better "picture of practice" of ambitious teaching in the mathematics classroom where effective integration of technology helped construct students' mathematical understanding.

13.1 Introduction

In education, we know that the value of any technology tool depends on how it is used in instruction. In this digital age, teachers are inundated with educational technology. It is important for teachers to be able to judiciously evaluate the instructional worth of a technology tool. Beyond the interactive dynamic nature of mathematics applets, educators need to ensure that the content taught using the virtual manipulative in an applet is characterized by ambitious teaching and learning goals. The term "intellectually ambitious teaching and learning," has been defined as instruction that helps students "develop in-depth knowledge of subject

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matter, gain higher-order thinking skills, construct new knowledge and understanding, and effectively apply knowledge to real-world situations" (Smylie and Wenzel 2006, p. 7). Ambitious teaching, according to Lampert et al. (2013), "challenge(s) teacher educators to prepare new teachers to do a kind of teaching that most experienced teachers are not yet doing" (p. 226). Furthermore, they suggest that the challenge of preparing beginning teachers in this way is asking novice teachers to teach in a way that is more socially and intellectually ambitious than the current norm. In addition, ambitious teaching using technology adds another layer of complexity because many pre-service teachers may not have a "picture of practice" from their own learning experiences.

To help pre-service teachers understand the complexity of ambitious teaching, educators and researchers have developed a set of key instructional activities that embody core teaching practices (Lampert et al. 2010). By "chunking" some of these instructional activities, teacher educators allow pre-service teachers access to "manageable, structured routines". These routines allow teachers to practice enacting a particular instructional purpose while maintaining the associated complexity. For example, in their study, Lampert et al. (2010) focused on the following four activities to promote ambitious mathematics teaching: choral counting, strategy sharing, computation strings, and solving word problems. Using this framework for designing instructional activities to help pre-service teachers manage ambitious mathematics teaching, this chapter presents three experiences for pre-service teachers focused on integrating technology in the mathematics classroom. These experiences were designed to move pre-service teachers along a spectrum from engaging with technology, because it seems appealing, to knowledgeably selecting virtual manipulatives for their conceptual development of mathematical content, higher-order thinking skills, and problem-solving ability. The aim of the chapter is to share the high level instructional activities that helped pre-service teachers integrate technology and promote ambitious teaching.

13.2 Understanding Research on Integrating Technology in the Content Area

According to Niess and Walker (2010):

> …many digital technologies have proved useful for students learning mathematics: graphing calculators, applets or virtual manipulatives, spreadsheets, computer algebra systems, and dynamic geometry tools. Each of these technologies provides visual representations that enable students to explore mathematical ideas in more dynamic ways. (Niess and Walker 2010, p. 100)

On the other hand, a misuse of technology would be using it as merely an attention grabber. Engagement is initially high when a lesson is introduced with dynamic and animated images, but soon that novelty wears off. According to the

Technology Principle in the Principles and Standards for School Mathematics (NCTM 2000), "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances student learning" (p. 24). The phrase "influences the mathematics that is taught" is what determines the ambitious teaching goals in the mathematics classroom. The word "enhances" is what characterizes technology as a tool with high leveraging power because technology has specific affordances that can enrich learning tasks (Suh 2010).

The complexity of teaching with technology stems from the notion that teaching in itself is a complex endeavor. Shulman (1986, 1987), coined the term Pedagogical Content Knowledge (PCK) to describe the specific knowledge needed to teach effectively which includes knowledge of subject matter, knowledge of students' thinking, and knowledge of pedagogy. In mathematics education, PCK has been expanded to include Mathematical Knowledge for Teaching (MKT) as the teacher "knowledge necessary to carry out the work of teaching mathematics" (Ball et al. 2008; Hill et al. 2005) that include specific high-leverage practices such as the use of mathematical explanations and representations, interpretations of student responses, and the ability to avoid mathematics errors and imprecision. Teaching with technology adds another layer of complexity to the PCK framework. Understanding how to teach with technology, referred to as Technological, Pedagogical, and Content Knowledge (TPACK) (Mishra and Koehler 2006) integrates a third component into teachers' specialized knowledge for teaching—the integration of technology into instruction. TPACK includes understanding how technology can be used to represent concepts, knowledge of pedagogical techniques that use technology to effectively teach content, familiarity with ways technology can help students understand particularly difficult topics, and knowing how technology can be used to build on existing knowledge. Virtual manipulatives have been described as an "interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer et al. 2002, p. 373). The questions is how to support pre-service teachers as they take advantage of technology to elicit sense making while students construct mathematical meaning.

In working with pre-service teachers (PSTs) using the virtual manipulative environment, it is important that they situate their learning within the current thinking around TPACK. Thus, prospective elementary mathematics teachers must be prepared to teach with and through technology because of the ways in which technology enhances mathematics learning, supports effective mathematics teaching, and influences mathematical content. Due to the complex nature of teaching with technology, Mishra and Koehler (2006) recommend that understanding approaches to successful technology integration requires educators to develop new ways of comprehending and accommodating this complexity. This is the challenge I have undertaken for this study.

13.3 Motivation for the Study

The purpose of this study was to carry out design-based research producing an improved course experience for teaching PSTs technology integration. It also allowed me to engage in a faculty self-study (Samaras 2010) as I reflected on previous courses and considered better ways to help PSTs learn how to integrate technology in a mathematics classroom effectively. For these reasons, I designed a series of practice-based activities that would give PSTs opportunities to work with mathematics applets while planning for a mathematics lesson. In this way, they would be able to refine their views of effective mathematics teaching, and develop a critical lens for their own practices, while building their repertoire for teaching mathematics with technology.

13.4 Method

13.4.1 Participants and Research Questions

The participants were 26 PSTs in a mathematics methods course also enrolled in a technology integration course. The technology integration course was one credit and was taught in tandem with the three-credit mathematics methods course. The methods course included field experiences and took place the semester before their student teaching internship.

The study explored the following two research questions:

1. What technology pedagogical content knowledge was elicited by PSTs on each of the practice-based assignments?
2. How do the designed practice-based assignments, situated in the course and field experiences, better support the development of PSTs technology pedagogical content knowledge?

13.4.2 Research Design

This study used cycles of design-based research aimed at developing effective approaches for integrating technology in the mathematics classroom. The methodology in this chapter was consistent with aspects of design experiments (Brown 1992). Design-based research (DBR) was chosen because it allows for practitioner research when implementing interventions and uses an iterative analysis process in conceptualizing learning, instruction, curricular design and reform. An iterative cycle of the following steps was used: considering a framework for integrating technology in the mathematics classroom, developing a curricular design with practice-based assignments, implementing the course-based activities and analyzing feedback for improvement for the next cycle. The DBR (Design-Based Research Collective 2003) communicates relevant implications to practitioners and other educational designers. Design was the focus of the study in an effort to foster learning, create usable knowledge, and advance theories of learning and teaching using technology in the classroom.

Grounding professional learning in practice can provide teachers with opportunities to investigate authentic problems of practice and to develop knowledge and skills in the contexts of their use. I designed three related practiced-based assignments with an initial phase that included asking PSTs to observe current implementation of technology in a mathematics classroom and reflect on the current practices they observed before implementing any tasks on an electronic discussion board. The first Practice-based Assignment (PBA) #1 was called *Technology Applet and Website Evaluation*. The task sheet was modified from the *Elementary and Middle School Mathematics Field Experience Guide* (Bay-Williams and Van de Walle 2010) to include two reflective questions on how PSTs would use the applets in their classroom and how the applets promoted the Common Core Standards for Mathematical Practice (CCSS-M 2010). The second Practice-based Assignment (PBA) #2: *Sequencing Technology Applets to Reflect on Students Mathematics Learning Progressions* was designed to expose PSTs to the mathematics standards, teaching practices standards, and the learning progressions of mathematics standards. The third Practice-based Assignment (PBA) #3 was called *Planning and Integrating Technology in a Math Lesson*, which allowed PSTs to pull together what they had learned from the previous assignments to design a thoughtful mathematics lesson integrating technology. After these three PBAs, PSTs were asked to reflect on their views of integrating technology in a mathematics classroom on an electronic discussion board.

13.4.3 Data Sources

Data sources included the collection of assignments described above: Practice-based Assignment (PBA) #1 *Technology Applet and Website Evaluation*, that included two applet reviews that were one page each (see Fig. 13.1); Practice-based Assignment (PBA) #2 *Sequencing Technology Applets to Reflect on Students Mathematics Learning Progressions*, that included a one page response recording sheet (see Fig. 13.2); Practice-based Assignment (PBA) #3 *Lesson Planning and Integrating Technology in a Math Lesson,* that included a formal lesson plan with a written reflection on how the lesson went and how students responded to the lesson with screenshots of student work analyzed.

In addition, PSTs submitted an electronic discussion board entry where they reflected on integrating technology in a mathematics classroom during their field experience. These data sources were analyzed for emerging themes. I used the participants' assignments to further analyze their learning using the TPACK components for teachers' specialized knowledge for teaching—to examine how their assignments elicited specific ways technology can be used to represent

Title: NLVM Algebra Balance Scales: Negatives http://nlvm.usu.edu/en/nav/frames_asid_324_g_3_t_2.html?open=instructions&from=category_g_3_t_2.html ***Type of Tool:*** Virtual manipulative ***Grade Level:*** VA SOL 6.18 (enrichment), 7.13 and 7.14, 8.15(a) ***Math Content:*** algebra
Specific Topic: evaluating simple linear equations with one negative variable on both sides
Key Instructional Objectives: using virtual manipulatives as representations of positive and negative variables and integers to balance equations
Rating: (Rate each aspect of the tool with a 1-5, 1 being lowest and 5 being highest. When appropriate add a comment or your reasoning too.)

Criteria	Rating	Comments
The applet or website provides better opportunities to learn than alternative approaches.	5	This applet constrains the learner's use of manipulatives so s/he is not distracted by the physical objects.
Students will be engaged with the math content not the frills.	5	It is a simple layout, with a balance, operation buttons, and colored blocks that represent the variable *x*, coefficient, and constant. Students have solved the equation when only an *x* remains on one side of the equation – there is no feedback. Focus is on balancing the scale, with no distractors.
The applet provides opportunities for problem solving.	5	The user creates the equation and selects the order of procedures to solve the equation. After each step, a simplified equation is displayed below the original equation, which will help the learner develop an understanding that all the equations shown are equivalent.
The tool develops conceptual knowledge and supports student understanding of concepts.	5	Negatives, or opposites, are represented by red balloons that raise the side of the scale on which they are placed. Blue blocks and boxes represent positive variables and numbers that push down the scale. After creating their own equation, students place the manipulatives that represent the equation onto the scale. This visual allows students to better understand that they may perform any operation to solve, as long as they do the same thing to both sides of the equation to maintain balance. One drawback of this app is that it lacks the final step of substituting the *x* value back into the equation. Students benefit from closing the loop and proving why that value is a solution.
The tool develops procedural knowledge and supports student understanding of skills.	5	The student will refine strategy choice through practice and observation of the outcomes of each choice, as displayed in the equation box. This flexible approach to finding the answer supports procedural fluency.
The software or website allows the teachers to assess student learning through records and reports.	1	There is no data capability visible on the free version.
The Program is challenging for a wide range of skill levels.	5	This app builds upon a previous one that uses positive integers and variables. Challenge is limited to the student's creation of their own problems – there is no built-in feature to increase skill level.
The tool is equitable in its consideration of gender and culture.	5	This virtual manipulative app is free of any discrimination, and provides equitable access to the skills practice of balancing equations.
The tool promotes good student interaction and discussion.	2	It is a one-user app. The teacher would have to prompt student users to discuss their strategies and discoveries following the user of this tool.
The tool has quality supplementary materials such as blackline masters.	1	Blackline masters are not evident on the free version.
Your overall rating	5	

Fig. 13.1 Evaluation of an applet by a pre-service teacher

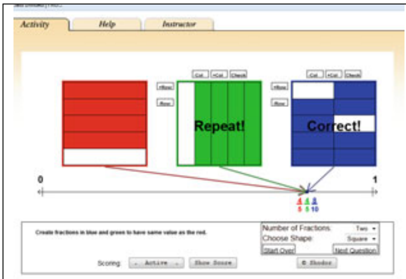
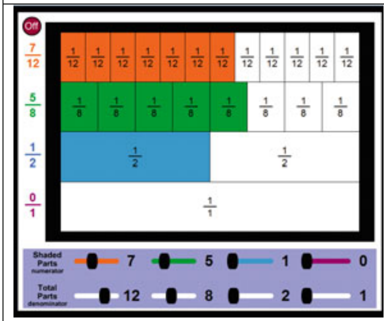
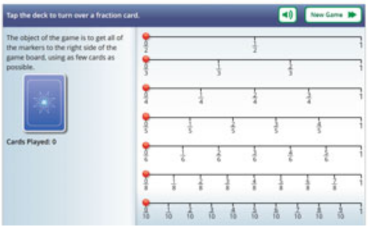
 Equivalent Fractions Finder http://www.shodor.org/interactivate/activities/EquivFractionFinder/	I selected this tool because my student struggled with fractional parts and because this tool allows the whole to be broken into fractional parts in several different ways. Students can manipulate the tool to create equivalent fractions in several different ways. I would encourage the student to create images as I have done below in the screenshot. Based upon my assessment, it seems the student is used to seeing fractional parts as the same size and shape. It's important that he understand the concept of fair shares. By creating images that are not what we normally see as fractional parts, it will deepen his understanding of this fraction concept. The student can save time by not having to color blocks repeatedly. Because the tool is online, students can divide the units in several different ways and they can do it multiple times.
 Fraction Bars http://www.mathplayground.com/Fraction_bars.html	My student seemed to have a great understanding of equivalent fractions. I chose this tool to challenge him and to move on from fraction concepts to adding and subtracting fractions with like and unlike denominators. Using this tool, students must add and subtract fractions with like and unlike denominators. So, it addresses the content well. Students must think critically about making the best choice for their move based upon their knowledge of equivalent fractions and the ability to move all tracks to one in the least amount of moves. I would use this tool to expose the students understanding of equivalent fractions by also having him keep a journal of why he chose to make each move.
 Fraction Tracks http://illuminations.nctm.org/Activity.aspx?id=4148	My student seemed to have a great understanding of equivalent fractions. I chose this tool to challenge him and to move on from fraction concepts to adding and subtracting fractions with like and unlike denominators. Using this tool, students must add and subtract fractions with like and unlike denominators. So, it addresses the content well. Students must think critically about making the best choice for their move based upon their knowledge of equivalent fractions and the ability to move all tracks to one in the least amount of moves. I would use this tool to expose the students understanding of equivalent fractions by also having him keep a journal of why he chose to make each move. The online fraction tracks game has an advantage over the paper version of the game because it can correct your move in real time. The program will not allow a student to make an incorrect move. If playing the two person version, the game also keeps track of which player's turn it will be.

Fig. 13.2 Selected applets to assess students learning progression

concepts and the specific knowledge of pedagogical techniques to effectively teach content. These data sources also provided feedback for ways to improve the designed experiences for integrating technology in the mathematics classroom. PSTs' responses from the discussion board on their beliefs about technology integration in the mathematics classroom were collected prior to and after the series of practice-based assignments to document how their views evolved after implementing these assignments. The responses to each of the three assignments also served as a way to analyze how the practice-based assignments were helping PSTs learn specific TPACK.

13.4.4 Data Analysis

Using Mishra and Koehler's description of TPACK (2006), I analyzed the learning that occurred through each assignment based on how the PSTs reflected on the knowledge of student thinking and learning, knowledge of content and instruction, and knowledge of technology. For the analysis process, I used a grounded theory approach to make sense of the data by defining codes, categories and concepts. Using the constant comparative method (Strauss and Corbin 1990), I began with open coding to develop names and categories. Then I moved to axial coding to relate the initial codes to one another. Finally, I applied selective coding to make choices on the most important codes. Using this method, I was able to analyze PSTs' assignments and reflections to display and organize categories in such a way that I was able to draw some concepts together as the emerging themes (Miles and Huberman 1994). I revisited the assignments and the reflections with the themes and categories the second time, to verify that the themes emerged across the multiple data sources.

13.5 Results

13.5.1 Initial Phase: Ideal Image of Practice and Observation in the Field

In order to help PSTs reflect on the current research on integrating technology, I asked them to read articles and the framework around TPACK. The major themes they drew from their reading were that technology is an essential tool for both learning and teaching mathematics and that the teacher's role is to select effective tools. In class, we discussed NCTM's (2000) position statement on the role of technology in teaching, which "regards technology as an *essential tool* for both learning and teaching mathematics" (p. 113).

The first reflection prompt posted for PSTs engaged them in a discussion about what they were able to observe in terms of technology integration in the mathematics classroom at their field sites. The prompt stated, "Now that you are visiting your classrooms, have you seen some ways 'real' teachers are integrating technology, particularly in the mathematics classroom? If so, share some great examples. What are some technologies available for your teachers and students? If you do not see technology being integrated in the mathematics classroom, in what ways would you integrate technology in what students are learning in your assigned classrooms?"

Common themes were the use of technology in the mathematics classroom including: (a) *focusing on visual representations*; (b) *using dynamic features to illustrate mathematics concepts; and*, (c) *incorporating collaborative learning centers.* The following is an excerpt from one of the discussion posts that illustrated the use of technology as a visual representation.

> The classroom teacher would create graphic representations for fractions where the students could come up and color sections to show their thinking. These were usually whole group lessons and students were engaged and active in working through the problems and discussing their solutions.

Another example was how the teacher used a virtual hundreds chart to keep track of patterns to facilitate a mathematics discussion about number patterns. The pre-service teacher noted how the teacher engaged students in analyzing the patterns and not just saying the numbers that were highlighted.

> The teacher pulled up a virtual 100s chart on the board, where the numbers could change color once she touched them. She used the chart to give a visual representation of counting by tens starting at various numbers on the chart. First they counted by tens starting at ten, and after every correct answer the teacher touched that number on the screen so it changed color. Once the "red tower", as the children called it, was completed on the chart, she asked the students to take a closer look at it and explain what they notice about the "red numbers" (analyzing the numbers in the ten's place and the one's place). Next she asked them to count by tens from 6. She, again, touched the number on the screen after every correct answer until another "red tower" appeared. The students really enjoyed comparing the two "red towers" and analyzing the "red numbers".

One pre-service teacher admitted to seeing benefits of virtual manipulatives despite her preference to use physical manipulatives, noticing the value of virtual base-ten blocks. She observed first graders working on a video game website and voiced how she would rather have them practice on the virtual manipulatives base ten website.

> While I prefer the hands-on nature of physical manipulatives, I can see the benefit of virtual ones. The base-10 blocks are the best and most frequent example of the virtual manipulative; these virtual blocks can show the student very effectively how to add, subtract, multiply and divide, the four key operations on which all other math is built. In the first grade class where I am currently observing I would use the base-10 blocks as a starting point rather than having the students practice using the video game website as they do now.

In addition to whole group lessons observed, several PSTs posted responses on the discussion board about ways technology allowed for collaboration and how teachers used technology as a center during guided mathematics lessons.

> The teacher mainly used the SMART board when conducting her lessons, but the students loved being able to come up to the board and manipulate the objects on the screen. Many times the teacher would put problems on the board and the students could come up and uncover the correct answers. The teacher also conducted guided math, so the students had

math centers. During the math centers, the students had the option of using to computers to play math games related to the topics being covered that week.

Some concerns and challenges were also shared in the postings. Some of the themes ranged from the *inequity of resources, anxiety caused by failure of technology, and apprehension about classroom management when using virtual manipulatives*.

Inequity of resources

In the school that I am placed in it is a Title 1 school, so from what I observed there are no computers, iPad, SMART boards, or anything in the classroom. The classroom did have a television that the teacher had hooked up to her own computer and that was the only technology I saw. From what I observed I did not see any carts that had iPad or computers on them for the teacher to reserve them. If the teacher can reserve computers the way I would use them in the classroom is by using various apps to aid students learning.

Anxiety caused by failure of technology

I have to admit that I am feeling some anxiety in using technology in one of our lesson plans. I think I feel like there is always the possibility of the computer crashing, the Internet not working, or one of the many other errors that could occur turning our lesson into a total flop. Perhaps this is the same anxiety that some of our cooperating teachers feel. It's like they know that it is important to incorporate technology but they are not comfortable with its incorporation in the world of math quite yet.

Apprehensive about classroom management

My only concern with the use of virtual manipulatives during morning work and center work is keeping track of the students proper engagement with the games, since many students may take advantage of being in front of a tablet or computer to wander off to different websites, or to fake that they are engaged when they are actually not productively thinking. I really do not know if there is a way to control that. However, balancing the use of the games and virtual manipulatives with the traditional games and worksheets would make the routine more interesting and help students who need a variety of methods to learn.

These responses served as an initial baseline for the ways PSTs were thinking about technology integration in the mathematics classroom.

13.5.2 Practice-Based Assignment #1: Technology Evaluation-Selecting a Mathematics Applet/Technology

In the first assignment, when asked to evaluate applets, PSTs freely chose applets that were available on the web. They used the evaluation form (see Fig. 13.1) categorizing the websites as drill and practice, virtual manipulatives, or investigations. Then they reflected on two reflective questions asking how they would use the applets in their classroom and how the applets promoted the Common Core State Standards for Mathematical Practice (CCSS-M 2010). Using Smith and Stein (2011) Levels of Cognitive Demand, I sorted the tasks as low level and high level cognitive demand to evaluate the potential that the tasks had for mathematics connections (see Table 13.1). According to Smith and Stein (2011), low-level tasks are characterized

Table 13.1 Analysis of applets chosen by PSTs

Low-level cognitive demand 12/40	*Applets for memorization* Examples: rehearsal of math facts-addition, subtraction, multiplication and division games that rehearse math facts tied to race or sports awards 1. Money recognition applets 2. Identifying fractions (matching exercises) 3. Ghost number sequencing	*Procedures without connections applets* 1. Online addition arcade game 2. Multiplication fluency game 3. Rounding estimation game 4. Fraction number line game 5. Garage sale money game 6. Counting Apple game 7. Place value game 8. Add like mad 9. Turtle addition game
High-level cognitive demand 28/40	*VM applets for procedures with visual representations and connections* 1. Fractions—parts of a whole (NLVM) 2. Money on NLVM 3. Base ten addition on NLVM 4. Fraction multiplication NLVM 5. Fraction feud (calculationnation) 6. Thinking blocks—Ratios 7. Okta's rescue illuminations 8. Number line bounce NLVM 9. Spin the big wheel! (explorelearning) 10. NLVM algebra balance scales 11. Deep sea duel (illuminations) 12. Adaptedmind-math 13. Factor dazzle (calculationnation) 14. Simple maze game (Shodor) 15. Base blocks addition (NLVM) 16. Equivalent fractions (Shodor)	*Doing mathematics/problem solving/logic games websites* 1. Explore learning: cannon ball 2. Thinking blocks: modeling problems: 3. Game/puzzle-circle 21 4. Kenken 5. Illuminations-bobby bear 6. Mathport 7. Proportionland 8. Math by design 9. Scale city 10. Rock and roll roadtrip 11. Explorelearning: walk the line 12. Coin logic problem: NLVM
Open-ended tools: virtual manipulatives Environment for open exploration 3/3 1. Pattern blocks on NLVM 2. Tessellate on Shodor interactivate 3. Glencoe virtual manipulatives		

by memorization and procedures without connections and high-level tasks are described as procedures with connections and doing mathematics.

This sorting exercise made me reflect that, in the next iteration of this assignment, I would ask PSTs to categorize the applets further using the levels of cognitive demand as it helped make a distinction between tools that rehearsed procedures only and tools that help build procedural understanding and problem solving. An example of a low level applet was a game and review for math facts that was procedural without connections to conceptual development. In the Math Sport game, students had to answer multiplication facts accurately to get a chance to make free throws as a reward. However, the PST rated it high, giving it a 4 out of 5. Another applet that was rated 4 was a place value game where one would have to identify the place value by clicking on the numeral. There was no assessment of the value of the digits just a recognition of the place value name. Although the PSTs noted that it was just rehearsing facts, they felt like the games added a level of engagement with rewards that would encourage the development of fact fluency. Table 13.1 shows the collection of applets compiled from the PSTs that was sorted by the levels of cognitive demand.

13.5.2.1 Opportunities to Elicit TPCK and Redesign for PBA #1: Recognizing the Need to Consider the Cognitive Demand

From the collection of applet evaluations, it was clear that PSTs needed more tailored instruction on how to select mathematics applets that offered opportunities to extend students' mathematics thinking and learning. Although I was surprised that some drill and practice games were rated as high, most of the applets chosen by PSTs were high-level mathematics applets. When I used the levels of cognitive demand to sort the forty applets evaluated by the PSTs, I categorized 12 as low-level applets and 28 as high-level applets. Three others were put in a separate category called open-ended tools because the level of the task depended on how teachers implemented the activity using the virtual tool. By sorting through the applets and websites the PSTs reviewed, it was apparent that the PSTs were able to find high cognitive demand applets. However, to increase the rigor of this activity, the next iteration of this PBA will include the levels of cognitive demand as part of the criteria when having PSTs evaluate applets. Since the choice of technologies affords and constrains the types of concepts and processes that can be taught, using Stein and Smith's framework, I determined it would help PSTs think about the level of cognitive demand present in a task and encourage the goal of ambitious teaching.

The following analysis is of the work (Fig. 13.1) of one of the participants, Kathy, a PST who explored the NLVM Algebra Balance Scale. She was placed in an upper elementary mathematics classroom teaching advanced mathematics during her field experience.

Her overall rating was a 5 even though she rated a few criteria with ratings of 1 and 2. It was obvious why she rated it high after reading her reflection. When asked, "How would you use this tool to bring out the Standards for Mathematical Practices?" she cited three specific practices: (1) Make sense of problems and persevere in solving them; (2) Model with mathematics; and, (3) Look for and express regularity in repeated reasoning. This PST showed evidence cited by Mishra and Koehler's description of TPACK (2006), where she reflected on the knowledge of student thinking and learning, knowledge of subject matter, and knowledge of technology. Kathy detailed in her reflection how the applet encouraged the following mathematical practices.

1. Make sense of problems and persevere in solving them—As students write their own equations, they will need to persevere in using the tool so that they learn to do the same thing to both sides and maintain a balanced equation each time. Linking the number representations to the manipulatives on the balanced scale will help them organize and make sense of equations with variables as well as gain a stronger conception of negatives as opposites that lift or subtract weight from the balance.

2. Model with mathematics—This app should be used to supplement teaching of variables and balancing equations. One benefit of the app is that the user cannot click “continue” until the equation has been correctly represented with blocks and balloons on the scale. Also, users may evaluate as many equations as they wish to create, which eliminates the constraints of physical materials. Students will discover the “idea of a variable as something that varies” (Van de Walle et al. 2014, p. 118). To enhance this app, I would have students then prove their solution by substituting the x value back into the equation. This is an important step in students’ mathematical thinking—reflecting on and justifying their answers.
3. Look for and express regularity in repeated reasoning—Through the repeated exploration of this app, students will discover how strategies they choose work well or not. They will also link the correct manipulation of blocks and balloons to the values in their initial equation and each simplified equivalent. This will increase their fluency with variables and enhance their algebraic skills.

In this PBA, it was evident through the many other PSTs’ analyses that they were thinking deeply about the important role of technology tools in education and how technology can, as Goldenberg (2000) states, “help students develop new and powerful ways of looking at problems, help them build mental models, acquire generalizable and flexible skills” (p. 6). In Kathy’s reflection, she recommended improvements to the applet that would allow collaborative learning and mathematics communication:

> I would have a two-player feature, to encourage more positive social interaction as students discuss and justify their mathematical thinking and to include a printable report, so this added feature would enable teachers to collect and analyze data related to their learners’ practice and thinking.

For Practice-Based Assignment #1, the themes revealed that the PSTs were developing understanding that different technology tools had specific affordances that can help develop conceptual understanding as well as procedural understanding and important mathematical practices. The thoughtful responses from the pre-service teachers demonstrated evidence that they were taking a critical look at the applets with student learning at the center of their analysis.

13.5.3 Practice-Based Assignment #2: Sequencing Technology Applets to Reflect on the Mathematics Learning Progressions

The second Practice-based Assignment was designed so that PSTs could think deeply about the learning progressions for a specific mathematics concept and select three related applets that could be used to teach and learn that concept. This assignment was also related to two of their big field assignments where they had to plan and teach a lesson using technology and assess a student's understanding about a concept using a variety of representations. The instruction for the PSTs was the following:

> Locate three different virtual manipulatives or applets that support the mathematical content you will address in the student assessment project for EDCI 552. Using the template below, analyze the models you have selected and evaluate them on their effectiveness and fidelity to the mathematical concept.

The following analysis is of the work submitted by one of the participants, Cindy, who illustrates how she interpreted this assignment and the TPACK learning that was elicited from the activity (see Fig. 13.2). Cindy chose three fraction virtual manipulative applets and discussed the affordances of each of the tools and how it could help the students she assessed as part of her student assessment assignment.

13.5.3.1 Opportunities to Elicit TPCK Through PBA #2: Mapping Along the Mathematics Learning Progression

Cindy reflected on how the different tools have different affordances. All three applets selected offer a variety of representations including using a region model, area model, and a number line model tied to the symbolic representation of the fraction notation. The concept that she focused on appears in our state's 4th-grade standard, "The student will (a) compare and order fractions and mixed numbers; (b) represent equivalent fractions" and most closely aligns to the CCSS-M (2010) Numbers Grade 4 Fractions A.1 and A.2:

Extend understanding of fraction equivalence and ordering.

> <u>CCSS.MATH.CONTENT.4.NF.A.1</u>
> Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
> <u>CCSS.MATH.CONTENT.4.NF.A.2</u>
> Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

The first applet, the Equivalent Fraction applet, ties the pictorial representation of a region model with the symbolic notation of the fraction and places the fraction on the number line. This applet allows users to compare up to three fractions with different numerators and different denominators (e.g., by creating common denominators or numerators). The Fraction Bar, the second applet, has a customizable feature for changing the numerators and denominators for four different bars. Cindy used the second applet to help her students think about comparing fractions using benchmark fractions such as ½. The final applet, Fraction Track, allowed the student to demonstrate his understanding of the fraction as a number on the number line. Not only did Cindy find appropriate applets that aligned to the mathematical learning goal, but she was able to articulate why she chose the applet and what specific mathematics the tool would highlight for the learner. By selecting these three related applets, Cindy demonstrated an understanding of the concept of representing fraction on a number line, comparing fractions by renaming fractions with like denominators, and using the benchmark of ½ to compare.

Another important mathematical practice when learning to teach mathematics is the ability to understand learning progressions to appropriately assess students' understanding. The final assignment in the mathematics methods course was to assess students' understanding. After Cindy administered this individual assessment, she revisited the applets that she had sequenced using learning progressions to make recommendations for further instruction.

> While the student was able to come to a correct answer regarding the placement of $\frac{5}{8}$ and $\frac{7}{12}$ on the number line, it did take some prodding and additional questioning. One virtual manipulative that may be helpful to this student is Math Playground's Fraction Bars found here: http://www.mathplayground.com/Fraction_bars.html. It was difficult for the student to verbalize that $\frac{1}{12}$ over the one half was different and less than $\frac{1}{8}$ over one half. I would use this tool to first allow my student to visualize the fractions of $\frac{7}{12}$ and $\frac{5}{8}$. With manipulative practice to see the relative size of fractional parts combined with practice comparing closeness to ½, the student would be better equipped to compare $\frac{5}{8}$ and $\frac{7}{12}$.

Here Cindy clearly demonstrates her TPACK and notes that the tool affords an opportunity to help students visualize the benchmark ½ and use that to compare and place 5/8 and 7/12 along the number line connecting 4/8 and 6/12 as ½ and having the student make sense of the remaining 1/8 versus 1/12.

Another recommendation Cindy makes for her student is to expose him to multiple images for fractions partitioned in equal parts, which may not always be congruent regions, by using the area model and knowing equal area (see Fig. 13.3).

> When looking to the question where the child must select which images from a group of seven are correctly partitioned into fourths, we see that the child struggles with understanding fractional parts as equal area regardless of shape. Similarly, this child can work on

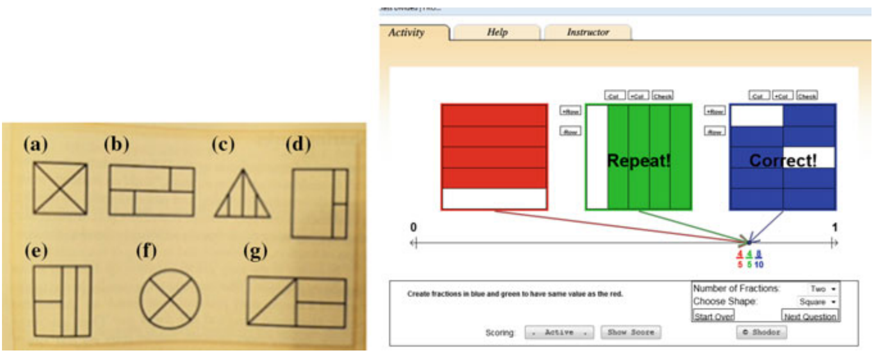


Fig. 13.3 Applets helped connect finding equivalent fractions with equal parts

> finding fair shares using the "Equivalent Fractions Finder" manipulative found here: http://www.shodor.org/interactivate/activities/EquivFractionFinder/. He can use this virtual manipulative to create images that are not normally seen as fractional parts in order to deepen his understanding of this fraction concept. In the image below, the "Equivalent Fractions Finder" manipulative was used to represent the same fraction in three different ways.

For Practice-Based Assignment #2, the themes revealed that the PSTs were developing the pedagogical strategies of using learning progressions of mathematics concepts to scaffold and tier the teaching and learning sequence.

13.5.4 Practice-Based Assignment #3: Integrating Technology in a Mathematics Lesson

In this final case study, a PST, Linda, planned a whole group lesson where students created patterns. The focus of the lesson was to recognize, describe, extend, and create a wide variety of growing and repeating patterns. She differentiated the tasks with parallel tasks using the Open Virtual Manipulatives site from Glencoe and the National Library of Virtual Manipulatives site (see Fig. 13.4). She began the lesson working with her class as a whole group to create ABAB, ABB, AAB, and ABC patterns. Each student completed and labeled a generated pattern. For an extra challenge, Linda prompted students to turn and talk with partners asking, "What will be the 17th (next) color? What will be the 20th color?"

Linda reflected that the important idea that she learned from this lesson was that conducting a rich and focused mathematics lesson requires one to be fully prepared with content knowledge and engaging materials that motivate students to learn. She also remarked that "the objectives, tasks, and assessment must all tie together for a cohesive, standards-based learning experience."

One aspect that Linda gave particular attention to was integrating technology to facilitate the equitable access to learning for all students by differentiating the tasks. She stated,

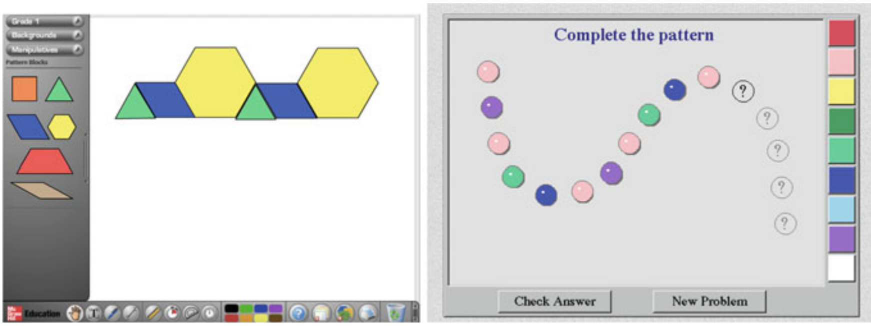


Fig. 13.4 Parallel tasks planned for a lesson on patterns

> An important skill that I must continue to work on is challenging students to develop higher-order thinking skills by posing analysis and synthesis problems. For example, students could generalize their learning about patterns through a project that helps them discover or generate patterns in their environment. Another way to stimulate higher-order thinking is to ask open-ended questions. I could pose a question such as, "How can knowing the core help you find out what comes next in the pattern?" This requires students to analyze and interpret what they know to discover the unknown. I could also include the patterns found in music, which would greatly enrich the lesson and add creativity. All of the applets used for guided practice were quicker and easier to use than paper and pencil, which allowed each student to have a turn with the technology and gave me the opportunity to probe their thinking.

13.5.5 Creating Their Own Practical Image of Practice After Performing in the Field: A Summary of PSTs' Learning

Goldenberg (2000) stated, "We must also provide time and opportunity for teachers to become fluent with the tools so that they can be flexible, use spur-of-the-moment good judgment in their classrooms, and not feel constrained by the tools or stilted by a lack of confidence in their ability to use them" (p. 7). After providing PSTs time and space to work with technology tools in the mathematics classroom, I was interested to examine how their own beliefs of teaching through technology may have evolved. In their final discussion prompt, I asked them: "On the discussion board, post your ideas for how you plan to incorporate technology into your math lessons. Cite Van de Walle's recommendations as you reflect on the implication to how you plan to integrate technology in your math lessons."

After their field experience, where PSTs planned and taught a mathematics lesson integrating technology, the most common theme was recognizing the need for more TPACK. This was a discussion that was not evident in the initial posting.

> Before I am able to effectively use digital tools for math instruction, I will seek out professional development opportunities to become more fluent in their functions and applications. Specifically, I need to strengthen my ability to use and teach using the graphing calculator. Its capabilities are many, like computing large quantities, applying a mathematical representation to model a real-world situation, testing a solution to check if it makes sense in context, estimating values to examine relationships, and selecting a strategy to solve a problem, to name just a few. I want to become more knowledgeable about its functions and teach it correctly so that students may use it confidently during their explorations of math concepts.

The finding that TPACK was the most common theme in their reflections was revealing because PSTs recognized that their knowledge of how to integrate technology was key.

Another pre-service teacher shared how she will need more time to familiarize herself with ways technology can help students understand particularly difficult topics and knowing how technology can be used to build on existing knowledge.

> I agree that there are so many useful tools out there and that it is our responsibility as educators to become experts in TPACK components. In order for us to assist students with exploring the latest technology tools we need to be able to navigate them first. I am looking forward to having a little bit of time this summer to explore more tools and also plan on taking advantage of any professional development opportunities that become available in order to build up my fluency in the available technology out there.

The importance of integrating technology in purposeful ways was voiced in this pre-service teacher's response as she commented on going beyond her technology apprehensions.

> I want to integrate technology when appropriate in my classroom and ensure that it is effective, not just incorporated to say it was. It should be beneficial to the objective of the lesson and not just an add-on to give the students something to do. It should have a well thought out purpose and be an integral part of the lesson. On the other hand, you also have to be prepared in the case that something happens and you cannot use the technology tools as we have all experienced at some point that some things just don't work out the way we have planned!

The belief that technology is a tool to bring equity to the classroom came up in this final response from one PST to another.

> I also agree with you that virtual tools help to bridge the achievement and economic gap when it comes to math content. What I mean by this is that free virtual manipulatives make math instruction equitable for all students. I did not make that connection in my post, but I am glad that you did so in yours. The fact that these tools are free, interactive and reinforce mathematic concepts is great (and truly valuable) for students in low socioeconomic schools. This is just another way to show that math can be fun and engaging when used in the right/appropriate way.

For Practice-Based Assignment #3, the themes revealed that the PSTs recognized the importance of understanding ways to use technology to provide more equity and access for diverse learners while helping students bridge a gap when they have specific learning difficulties in mathematics. This also related to their need to learn more about ways to build on students' existing knowledge.

Through these final responses, it was evident that PSTs were thinking more deeply about the importance of TPACK in their development and in their practice. As a result of the sequence of practice-based assignments (see Fig. 13.5), PSTs had more time and space to explore virtual manipulatives as learners themselves and then to use them as teachers in the classroom. The sequence of practice-based assignments allowed scaffolding needed for PSTs to explore, analyze and plan for virtual manipulatives to become an important tool for teaching and assessing student learning.

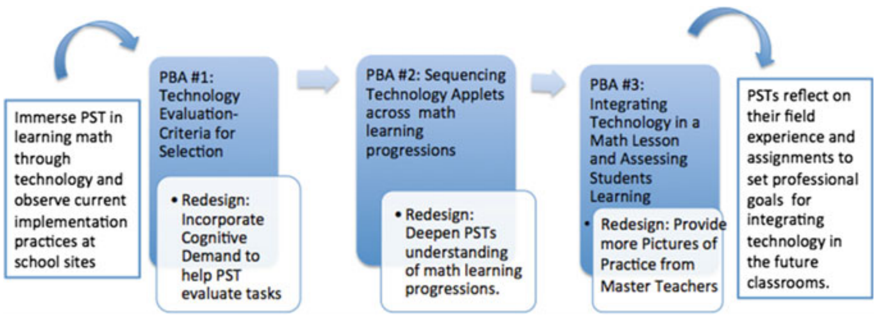


Fig. 13.5 Sequence of practice-based assignments

13.6 Conclusion and Consideration for Teacher Educators: Using Feedback from Assignments to Tailor the Next Cycle

The original purpose for the design of the practice-based assignments was to create manageable, structured routines that would allow PSTs to learn how to evaluate applets, select a few applets that aligned to their mathematics learning goals, and plan, teach, and assess student learning. The initial observation at their field site was designed so that they could have a "picture of practice" for integrating technology in the mathematics classroom. After reading and analyzing the first posts on the discussion board, it was clear that some PSTs observed best practices integrating technology while others did not. Although what they observed is the reality of today's classroom, as an instructor, I want to create an opportunity for PSTs to see several exemplars of best practices for integrating technology in the mathematics classroom. In this way, PSTs can have a "picture of practice" of ambitious teaching in the mathematics classroom that showcases effective integration of technology for learning. One way to create this opportunity is to plan instructional rounds to a classroom where the teacher is effectively integrating technology. By asking some master teachers to model instructional practice, the PSTs and I will have a collective experience of observing an exemplar lesson. Another way researchers and educators have offered this exemplary practice is through rehearsals. Lampert et al. (2013) share a method that they call rehearsal where the teacher educator and novice teachers conduct "run-throughs" or microteaching in methods courses. Here, the novice teacher teaches an instructional activity, while the teacher educator and other novice teachers are in the role of simulated classroom students, who "act back" in a way that students might in an actual classroom.

In addition to providing rehearsals and tailored observations with masterful teachers, I learned through *PBA #1: Technology Applet and Website Evaluation* that PSTs need to evaluate applets with cognitive demand in mind. The analysis I experienced when sorting through the PST selected applets allowed me to consider what level of cognitive demand the applets offered. Was the applet allowing for the user to make sense of a procedure with connections or was the applet offering opportunities for problem solving and doing mathematics? These are good ways to evaluate technology tools beyond examining external features and would forward ambitious teaching and learning goals. Another lesson I learned from *PBA #2 Sequencing Technology Applets to Reflect on the Mathematics Learning Progressions* was the need to sequence the variety of applets that are worthwhile using the mathematics learning progression more explicitly. PSTs were exceptional at aligning the applets to a standard, but more instruction could be provided to look across the vertical strands to map out the learning trajectory. This vertical articulation would be beneficial for PSTs as a thread for frequent classroom discussions when planning, designing and conducting diagnostic assessments for the course. Finally, I learned from *PBA #3 Integrating Technology in a Math Lesson* that PSTs may need more rehearsal with supportive co-teachers modeling ideal ways to integrate technology in the mathematics classroom. As mentioned in the introduction, using technology in the mathematics classroom may be a novel approach for many in-service teachers who may not be using technology in their mathematics classrooms. There is a need to work with both in-service teachers and pre-service teachers to develop their repertoire for integrating technology effectively in the mathematics classroom. Providing more "pictures of practice" from master teachers who can serve as exemplary models of ambitious teaching using technology in the mathematics classrooms may help build PSTs repertoire as they are enter the teaching profession.

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